Michael Janas,[*] Michael E. Cuffaro,[†] Michel Janssen[*]

Understanding Quantum Raffles

Quantum Mechanics on an Informational Approach: Structure and Interpretation

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[*]School of Physics and Astronomy, University of Minnesota; E-mail: jana0030@umn.edu, janss011@umn.edu

[†]Munich Center for Mathematical Philosophy, Ludwig-Maximilians-Universität München; E-mail: Michael.Cuffaro@lmu.de

We dedicate this volume to the memory of Itamar Pitowsky (1950–2010) and William Demopoulos (1943–2017).

In addition, Cuffaro dedicates his efforts to the memory of Giuseppe, his father (1932–2019) and Janssen dedicates his efforts to the memory of Heinrich, his father (1929–2015).

Foreword

Understanding Quantum Raffles was inspired by *Bananaworld*, as the authors say, but it is very much more than that. My initial aim in writing *Bananaworld* was to de-mystify quantum entanglement for non-physicists—as Schrödinger remarked, '*the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.' I wanted to show that entanglement is essentially a new sort of nonlocal correlation, explain why it is puzzling, and point out how it can be used as a resource. The device I used to exhibit entanglement was the Popescu-Rohrlich nonlocal box, or PR-box, which I dramatized as a pair of bananas that each acquires one of two possible tastes when peeled in one of two allowable ways, from the stem end or the top end. The PR-box correlation is a superquantum correlation but can be expressed quite simply, without the mathematical machinery of quantum mechanics. It has all the puzzling features of quantum entanglement and, with a little poetic license, can even be exploited to show how entanglement works to enable feats like quantum teleportation, unconditional security in quantum cryptography, and apparently exponential speed-up in quantum computation.

In spite of the bananas, the book did not turn out to be the sort of thing you could pick up and enjoy over a beer. So I wrote *Totally Random: Why Nobody Understands Quantum Mechanics* with my daughter, Tanya Bub. *Totally Random* deals with some of the topics discussed in *Bananaworld*, but in a way that's much more accessible and, we hoped, fun to read. We presented the book as 'a serious comic on entanglement'—serious because we felt that the general reader could come away with a real understanding of entanglement: what it is, what the patriarchs of quantum mechanics have said about it, and what you can do with it. The authors of *Understanding Quantum Raffles*—the three Mikes—have evidently also given a great deal of thought to pedagogical issues. While some of the discussion, notably Chapter 4, tackles advanced material, a major part of the book, especially Chapters 2 and 3, is clearly intended for the general reader, so if you want to understand what is really new and interesting about quantum mechanics, this is the book to read.

In *Bananaworld*, I brought out the difference between classical and quantum mechanics by considering to what extent it is possible to simulate a PR-box correlation with various resources, classical or quantum. Bell's nonlocality proof amounts to a demonstration that two separated agents, Alice and Bob, restricted to classical, and so local resources (effectively what computer scientists call 'shared randomness'), can achieve an optimal success rate of no more than 75%. If Alice and Bob are allowed to use quantum resources, entangled pairs of photons or electrons, they can do better, about 85%. Equipped with PR-boxes, they can, of course, achieve a 100% success rate. Another way to put this is in terms of the Clauser-Horne-Shimony-Holt (CHSH) inequality for two bivalent Alice-observables and two bivalent Bob-observables. The CHSH correlation for the four pairs of observables is constrained to values between -2 and 2 for local classical correlations, between $-2\sqrt{2}$ and $2\sqrt{2}$ for quantum correlations, and between -4 and 4 for PR-box correlations, which are maximal for correlations that do not allow instantaneous signaling. Geometrically, as Pitowsky showed,[1] the classical or local correlations for this case can be represented by the points in an 8-dimensional polytope with facets characterized by the CHSH inequality and similar inequalities, the quantum correlations by the points in a convex set that includes the polytope, and the no-signaling correlations by a polytope that includes the quantum convex set.

The three Mikes do something brilliantly different. Instead of the CHSH inequality, they consider the Mermin inequality for three bivalent observables for each agent. In terms of bananas, Alice and Bob peel their bananas in one of three possible ways associated with three directions in which they are required to hold their bananas while peeling. This complication, which I blush to admit I first thought was pointless, results in a tetrahedron for the classical or local correlations, an elliptope for the quantum convex set (a 'fat' tetrahedron that includes the classical tetrahedron), and a cube for the no-signaling correlations—easily visualizable in three dimensions. The three Mikes produce two derivations for the non-linear inequality characterizing the elliptope: a derivation 'from within' quantum mechanics, which uses the Born rule for probabilities, and a derivation 'from without,' which follows work by Yule in the late 19th century on Pearson correlation coefficients. In Yule's derivation, the inequality is a general constraint on correlations between three random variables. In the 'proof from without,' the random variables are the eigenvalues of Hilbert space operators representing observables and the

[1] I. Pitowsky, 'On the geometry of quantum correlations,' *Physical Review* A 77, 062109 (2008).

constraint follows quite generally, without assuming the Born rule for quantum probabilities.

The Mermin inequality refers to spin-1/2 particles in the singlet state. Remarkably, it turns out that singlet state quantum correlations are confined to the elliptope even for higher spin values, while the tetrahedron for local classical correlations is replaced by a succession of polyhedra with more and more facets for higher spins, approaching the elliptope in the limit of infinite spin. All this is beautifully illustrated in 3-dimensional visualizations. The analysis is particularly impressive because it shows clearly and precisely how classical and quantum correlations are related in this particular case.

This is certainly the first book in which the word 'Bubism' appears. The three Mikes use the term to refer to 'an interpretation of quantum mechanics along the lines of *Bananaworld*, belonging to the same lineage, or so we will argue, as the much-maligned Copenhagen interpretation.' *Bananaworld* began as a discussion of entanglement, but as I wrote the book it evolved into a way of thinking about the transition from classical to quantum mechanics. The three Mikes have taken this perspective and articulated and developed it into an interpretation that I fully endorse but which owes as much to their careful analysis of the conceptual issues as my own thinking.

I added the last chapter to *Bananaworld*, 'Making Sense of it All,' because I thought I should say something about the measurement problem of quantum mechanics as it is usually understood, and how various interpretations propose to solve the problem. But the chapter doesn't fit well with the rest of the book, which, taken as a whole, was already an attempt to make sense of it all. The revised version in the paperback edition is an improvement, but not entirely satisfactory. Chapter 6 of *Understanding Quantum Raffles*, on interpreting quantum mechanics, nails it.

Here, following the account by the three Mikes, is how I now see the view they call Bubism. Quantum mechanics began with Heisenberg's unprecedented move to 'reinterpret' classical quantities like position and momentum as non-commutative. In a commutative algebra, the 2-valued quantities, representing propositions that can be true or false, form a Boolean algebra. A Boolean algebra is isomorphic to a set of subsets of a set, with the Boolean operations corresponding to the union, intersection, and complement of sets. The conceptual significance of Heisenberg's proposal lies in replacing the Boolean algebra of subsets of classical phase space, where the points represent classical states and subsets represent ranges of values of dynamical variables, with a non-Boolean algebra. Later, following the Born-Heisenberg-Jordan *Dreimännerarbeit* and further developments by Dirac, Jordan, and von Neumann, this non-Boolean algebra was formalized as the algebra of closed subspaces of Hilbert space, a vector space over the complex numbers, or equivalently a projective geometry. So the transition from classical to quantum mechanics is, formally, the transition from a Boolean algebra of subsets of a set to a non-Boolean algebra of subspaces of a vector space.

In his 1862 work ‘On the Theory of Probabilities,’ George Boole characterized a Boolean algebra as capturing ‘the conditions of possible experience.’ Classical theories are Boolean theories. The non-Boolean algebra of quantum mechanics (for Hilbert spaces of more than two dimensions) can be pictured as a family of Boolean algebras that are ‘intertwined,’ to use Gleason’s term,[2] or ‘pasted together,’ in such a way that the whole family can’t be embedded into a single Boolean algebra.[3] So in a quantum theory, the single Boolean algebra of a classical theory is replaced by a family of Boolean algebras, in effect, a family of Boolean perspectives or Boolean frames associated with different incompatible measurement experiences. The upshot, as von Neumann pointed out, is that quantum probabilities are ‘perfectly new and *sui generis* aspects of physical reality’[4] and ‘uniquely given from the start.’

The sense in which quantum probabilities are ‘uniquely given from the start’ is explained in an address by von Neumann on ‘unsolved problems in mathematics’ to an international congress of mathematicians in Amsterdam, September 2–9, 1954.[5] Here is the relevant passage:

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[2] A. N. Gleason, ‘Measures on the closed subspaces of Hilbert space,’ *Journal of Mathematics and Mechanics* 6, 885–893 (1957). The term is used to refer to intertwined orthonormal sets, which are Boolean algebras, on p. 886.

[3] Kochen and Specker proved non-embeddability for the ‘partial Boolean algebra’ of subspaces of a Hilbert space of more than two dimensions in S. Kochen and E.P. Specker, ‘On the problem of hidden variables in quantum mechanics,’ *Journal of Mathematics and Mechanics* 17, 59–87 (1967). Bell proved a related result as a corollary to Gleason’s theorem in J.S. Bell, ‘On the problem of hidden variables in quantum mechanics,’ *Reviews of Modern Physics* 38, 447–452 (1966), reprinted in J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).

[4] From an unpublished manuscript ‘Quantum logics (strict- and probability-logics),’ reviewed in A.H. Taub in *John von Neumann: Collected Works* (Macmillan, New York, 1962), volume 4, pp. 195–197.

[5] In Miklós Rédei and Michael Stöltzner (eds.), *John von Neumann and the Foundations of Quantum Mechanics*, pp. 231–246 (Kluwer Academic Publishers, Dordrecht, 2001). The quoted passage is on pp, 244–245. Also quoted (without the last sentence) in M. Rédei, “‘Unsolved Problems in Mathematics’ J. von Neumann’s address to the International Congress of Mathematicians Amsterdam, September 2–9, 1954,’ *The Mathematical Intelligencer* 21, 7–12 (1999).

> Essentially if a state of a system is given by one vector, the transition probability in another state is the inner product of the two which is the square of the cosine of the angle between them [sic].[6] In other words, probability corresponds precisely to introducing the angles geometrically. Furthermore, there is only one way to introduce it. The more so because in the quantum mechanical machinery the negation of a statement, so the negation of a statement which is represented by a linear set of vectors, corresponds to the orthogonal complement of this linear space. And therefore, as soon as you have introduced into the projective geometry the ordinary machinery of logics, you must have introduced the concept of orthogonality. This actually is rigorously true and any axiomatic elaboration of the subject bears it out. So in order to have logics you need in this set a projective geometry with a concept of orthogonality in it.
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> In order to have probability all you need is a concept of all angles, I mean angles other than 90°. Now it is perfectly quite true that in geometry, as soon as you can define the right angle, you can define all angles. Another way to put it is that if you take the case of an orthogonal space, those mappings of this space on itself, which leave orthogonality intact, leave all angles intact, in other words, in those systems which can be used as models of the logical background for quantum theory, it is true that as soon as all the ordinary concepts of logic are fixed under some isomorphic transformation, all of probability theory is already fixed.
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> What I now say is not more profound than saying that the concept of a priori probability in quantum mechanics is uniquely given from the start.

In *Bananaworld*, I defended what I called an 'information-theoretic' interpretation of quantum mechanics. The term is perhaps unfortunate. In the first place, it invites objections like those by Bell: '*Whose* information? Information about *what*?'[7] In the second place, the emphasis should be on probability, as the three Mikes make clear, with the understanding that information theory is a branch of probability theory specifically concerned with probabilistic correlations.

If relativity is about space and time, quantum mechanics is *about probability*, in the sense that quantum probabilities are '*sui generis*' and 'uniquely given from the start' as an aspect of the kinematic structure of the theory and are not imposed from outside as a measure of ignorance, as in classical theories, where probability is a measure over phase space. In this new framework, new sorts of nonlocal probabilistic correlations associated with entanglement are possible, which makes quantum information fundamentally different from

[6] Von Neumann evidently meant to say that the transition probability is the square of the (absolute value of) the inner product, which is the square of the cosine of the angle between them.

[7] J.S. Bell, 'Against measurement,' in *Physics World* 8, 33–40 (1990). The comment is on p. 34.

classical information. In a Boolean theory such correlations are impossible without introducing what Einstein called 'spooky' action at a distance.

Quantum probabilities are revealed in measurement, and a measurement is associated with the selection of a particular Boolean frame in the family of Boolean algebras that 'captures the conditions of possible experience.' In terms of observables, a measurement involves the selection of a basis of commuting observables in Hilbert space. As a consequence, the observer is no longer 'detached,' unlike the observer in classical mechanics, as Pauli observed.[8] The measurement outcome is a random assignment of truth values to the elements in the Boolean frame, or a random assignment of values to the observables in the corresponding basis. What's puzzling, from a Boolean perspective, is that measurement in a non-Boolean theory is not passive—not just 'looking' and registering what's there in a passive sense. Measurement must produce a change in the description, and that's not how we are used to thinking of measurement in a Boolean theory. Here's how Schrödinger puts it:[9]

> (1) The discontinuity of the expectation-catalog [the quantum pure state] due to measurement is *unavoidable*, for if measurement is to retain any meaning at all then the *measured value*, from a good measurement, *must* obtain. (2) The discontinuous change is certainly *not* governed by the otherwise valid causal law, since it depends on the measured value, which is not predetermined. (3) The change also definitely includes (because of 'maximality' [the 'completeness' of the quantum pure state]) some *loss* of knowledge, but knowledge cannot be lost, and so the object *must* change—*both* along with the discontinuous changes and *also*, during these changes, in an unforeseen, *different* way.

Quantum probabilities don't simply represent ignorance about what is the case. Rather, they represent a new sort of ignorance about something that doesn't yet have a truth value, something that simply isn't one way or the other before we measure, something that requires us to act and do something that we call a measurement before nature supplies a truth value—and removes the truth values of incompatible propositions that don't belong to the same Boolean frame, associated with observables that don't commute with the measured observable.

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[8] M. Born, *The Born-Einstein Correspondence* (Walker and Co., London, 1971). Pauli talks about the classical ideal of the 'detached observer' in a letter to Born dated March 30, 1954 on p. 218.

[9] 'Die gegenwärtige Situation in der Quantenmechanik,' *Die Naturwissenschaften* 48, 807–812; 49, 823–828, 844–849 (1935). The quotation is from p. 826. The translation is by John Trimmer, *Proceedings of the American Philosophical Society* 124, 323—338 (1980).

Schrödinger calls the measurement problem 'the most difficult and most interesting point of the theory.'[10] As the three Mikes aptly put it, the measurement problem is *a feature of quantum mechanics* as a non-Boolean theory, *not a bug*.

Interpretations of quantum mechanics that oppose the Copenhagen interpretation begin with Schrödinger's wave theory as conceptually fundamental, rather than Heisenberg's algebraic formulation of quantum mechanics, and propose dynamical solutions to what then seems to be a problem: how does what we do when we perform a measurement by manipulating some hardware in a laboratory select a Boolean frame in Hilbert space, a basis of observables that have definite values, and what explains the particular assignment of truth values to the elements in the Boolean frame, or the particular assignment of values to observables.

Bohm's theory tells a one-world Boolean story: position in configuration space is always definite, associated with a Boolean algebra, and other quantities become definite through correlation with position via the measurement dynamics. The problem here, as Bell showed, is that Bohm's theory is nonlocal in configuration space, allowing instantaneous action at a distance, which Einstein regarded as 'spooky'[11] and so non-physical (although averaging over the Born distribution hides the nonlocality). I suspect that it was for this reason that Einstein dismissed Bohm's theory as 'too cheap for me' in a letter to Born.[12]

The Everett interpretation tells a multi-world Boolean story in which everything that can happen does happen in some Boolean world. This avoids having to explain why *this* measurement outcome rather than *that* measurement outcome, since every possible outcome actually occurs in some world. The trick is to show how this fits Schrödinger's wave theory of quantum mechanics. There is no spooky action at a distance in the Everettian interpretation, but the measurement problem appears as the basis problem: how to explain the selection of a particular basis with respect to which the multiplicity associated with 'splitting into many worlds' occurs in a measurement process. Everettians solve the basis problem by appealing to the dynamics of environmental decoherence: as the environment becomes increasingly entangled with the measuring ap-

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[10] *ibid.*, p. 826.

[11] M. Born, *op. cit.*. The term is used in a letter from Einstein to Born dated March 3, 1947 on p. 158.

[12] M. Born, *op. cit.* The comment is on p. 192 in a letter from Einstein to Born dated May 12, 1952.

paratus, it becomes more and more difficult, but not in principle impossible, to distinguish an entangled state from the corresponding mixture with respect to a particular coarse-grained basis. Quantum probabilities with respect to the elements of this basis are explained in terms of the decision theory of an agent-in-a-world about to make a measurement. Even granting decoherence as an effective solution to the basis problem, it seems contrived to interpret the 'perfectly new and *sui generis* aspects of physical reality,' the Hilbert space probabilities that are 'uniquely given from the start,' in this way.

Understanding Quantum Raffles is likely to be a classic in the foundational literature on quantum mechanics. The three Mikes have produced an exceptionally lucid book on quantum foundations that is also suitable for readers, with some tolerance for basic algebra and geometry, who are looking for answers to conceptual questions that are typically glossed over in standard courses on quantum mechanics.

Jeffrey Bub
Philosophy Department
Institute for Physical Science and Technology
Joint Center for Quantum Information and Computer Science
University of Maryland, College Park

Preface

The volume you just got yourself entangled with was inspired by Jeffrey Bub's (2016) *Bananaworld: Quantum Mechanics for Primates*. Our original plan had been to contribute an article to a special issue of the journal *Studies in History and Philosophy of Modern Physics* devoted to Jeff's book. That article eventually grew and morphed into this monograph, which we feel can now stand on its own feet. We are proud to present it as a volume in the series *Boston Studies in the Philosophy and History of Science*. In this volume, on the basis of some novel technical results (Chapters 2–5), we present and defend an informational interpretation of the basic framework of quantum mechanics (Chapters 1, 6–7). Our primary target audience for this book is physicists, philosophers of physics and students in these areas interested in the foundations of quantum mechanics. However, in the spirit of *Bananaworld* and its sequel, the graphic novel *Totally Random: Why Nobody Understands Quantum Mechanics* written by Jeff and his daughter Tanya (Bub & Bub, 2018), we wrote parts of our book (especially Chapter 2 and Sections 3.1–3.2) with the idea that they could be used as the basis for courses introducing non-physics majors to quantum mechanics, or for self-study by those outside of a university setting with an interest in quantum mechanics. Such readers, however, should be prepared to brush up on some high-school mathematics along the way (basic algebra and geometry; sines and cosines; vectors, matrices and determinants—but absolutely no calculus).[13] We hope that all readers, even those who disagree with us on the basic issue of how their entanglement with our book results in them forming a definite view of its contents, will find something of value between its covers. This preface serves two purposes. First, we will briefly describe the contents of this volume. Second, we will give a brief history of how we came to write it, which will also give us an opportunity to thank the many people who helped us along the way.

Let us begin then by laying out the overall argumentative strategy of our book (which is in broad outline the same as it was in our original plan for a paper). We use correlation arrays, the workhorse of *Bananaworld*, to analyze the correlations found in an experimental setup due to David Mermin (1981) for measurements on pairs of spin-$\frac{1}{2}$ particles in the singlet state. Adopting an approach pioneered by Itamar Pitowsky (1989b) and promoted in *Banana-*

[13] See Section 2.6.2, note 28, for some recommendations for non-expert readers looking for introductions to the basic formalism of quantum mechanics.

world, we geometrically represent the class of correlations allowed by quantum mechanics in this setup as an elliptope in a non-signaling cube, which represents the broader class of all correlations that cannot be used for the purpose of sending signals traveling faster than the speed of light. To determine which of these quantum correlations are allowed by so-called local hidden-variable theories, we investigate which ones we can simulate using raffles with baskets of tickets that have the outcomes for all combinations of measurement settings printed on them. The class of correlations found this way can be represented geometrically by a tetrahedron contained within the elliptope. We use the same Bub-Pitowsky framework to analyze a generalization of the Mermin setup for measurements on pairs of particles with higher spin in the singlet state. The class of correlations allowed by quantum mechanics in this case is still represented by the elliptope; the subclass of those whose main features can be simulated with our raffles can be represented by polyhedra that, with increasing spin, have more and more vertices and facets and get closer and closer to the elliptope.

We use these results to advocate for Bubism (not to be confused with QBism), an interpretation of quantum mechanics along the lines of *Bananaworld*, belonging to the same lineage, or so we will argue, as the much-maligned Copenhagen interpretation. Probabilities and expectation values are primary in this interpretation. They are determined by inner products of state vectors in Hilbert space. State vectors do not themselves represent what is real in quantum mechanics. Instead the state vector gives a family of probability distributions over the values of subsets of observables, which do not add up to one overarching joint probability distribution over the values of all observables. As in classical theory, these values (along with the values of non-dynamical quantities such as charge or spin) represent what is real in the quantum world. Hilbert space puts constraints on possible combinations of such values, just as Minkowski space-time puts constraints on possible spatio-temporal constellations of events. To illustrate how generic such constraints are, we show that the one derived in this volume, the elliptope inequality, is a general constraint on correlation coefficients, which can already be found in much older literature on statistics and probability theory. Udny Yule (1897) already stated the constraint. Bruno de Finetti (1937) already gave it a geometrical interpretation sharing important features with its interpretation in Hilbert space.

As this brief synopsis shows, polytopes and philosophy form two pillars of this volume. The third pillar is pedagogy. As noted above, we wrote parts of this volume as an introduction to quantum mechanics for non-specialists. For many years, one of us (Janssen) used a combination of the paper by Mermin (1981) mentioned above and chapters from David Albert's *Quantum Mechanics and Experience* (Albert, 1992) to introduce quantum mechanics to non-physics majors in college and in high-school physics classes. Over the past few years, Janssen (assisted by Janas) has been developing a different approach, informed by and informing the material presented in this book. Like Albert (1992, Ch. 1, pp. 1–16, "Superposition"), we start, in Chapter 2, with certain stochastic experiments and show that classical theory (more precisely: local hidden-variable theories) cannot account for the statistics found in these experiments. Following Mermin rather than Albert, however, we choose (variations on) an experiment highlighting *entanglement* rather than *superposition* as the key feature that distinguishes quantum theory from classical theory (cf. Chapter 2, note 2 and Chapter 6, note 44). Albert (1992, Ch. 2, pp. 17–60) proceeds to give a concise and elementary exposition of the formalism of quantum mechanics (which we highly recommend to readers unfamiliar with it) and shows how it can account for the puzzling statistics presented in the opening chapter of his book. Yet it remains unclear how anybody would come up with this way of accounting for these puzzling statistics in the first place. Bub's *Bananaworld*, especially the notion of correlation arrays, allows us to do better. The correlation arrays for the puzzling statistics we start from can be parametrized by the sines and cosines of certain angles. In quantum mechanics such sines and cosines naturally emerge as components of vectors in various bases in what is called a Hilbert space. In Section 2.6, we introduce just enough formalism to get this basic idea across to non-specialists. More rigorous and more general versions of the arguments in Chapter 2 will be given in Chapter 4, which the reader can skip or skim (along with Chapter 5) without losing the thread of the overall argument (but we hope the reader will at least take a look at the pictures of correlation polyhedra in Figures 4.11, 4.13 and 4.17). The connection between quantum mechanics and general statistics and probability theory will be explored further in Chapter 3, also accessible to non-specialists with the exception of the later parts of Section 3.4. The upshot of Chapters 2–5 is summarized at the beginning of Chapter 6, making that chapter largely self-contained and thus suitable, all by itself, for courses on the foundations of quantum mechanics.

Polytopes, philosophy and pedagogy are the main interests of Janas, Cuffaro and Janssen, respectively. Accordingly, even though all three of us made substantial contributions to all seven chapters, Janssen had final responsibility for Chapters 1–2, Janas for Chapters 3–5 and Cuffaro for Chapters 6–7. The three of us came to this project from different directions. Janssen, a historian of science, is a recovering Everettian who has been defending Bub's information-theoretic interpretation with the zeal of the converted. Cuffaro, a philosopher of science, was and is mainly interested in quantum computation and information, but began to think seriously again about the interpretation of quantum mechanics through conversations with Bill Demopoulos before meeting Janssen in 2017. Janas, a theoretical physicist, was and remains a Bohm sympathizer. Though we each have our own unique interests and histories, one thing the three of us share is a broadly Kantian outlook, something careful readers familiar with that outlook will not fail to notice as they go through the pages of this volume.

This project started in the Fall of 2016 when, at Janssen's suggestion, the Physics Interest Group (PIG) of the Minnesota Center for Philosophy of Science of the University of Minnesota, devoted most of its biweekly meetings that semester to *Bananaworld*. This book rekindled Janssen's interest in Bub and Pitowsky's heretical contribution to the Everett@50 conference in Oxford in 2007, "Two dogmas about quantum mechanics" (Bub & Pitowsky, 2010). In these PIG sessions, Janssen presented his reworking of Mermin's setup for testing a Bell inequality in terms of Bub's correlation arrays along with a (clumsy) derivation of the so-called Tsirelson bound for this setup. Janas attended these sessions. On a return visit to *Bananaworld* in the Fall of 2017, Janas began to explore the geometrical representation of correlation arrays by polyhedra and polytopes. He thereupon joined Janssen and Cuffaro, who, at the 2017 edition of the conference New Directions in the Foundations of Physics in Tarquinia, had decided to write a response to *Bananaworld* together. In the Fall of 2017, Janssen gave a physics colloquium at Minnesota State University Mankato on our joint project, and then a lunchtime talk at the Center for Philosophy of Science at the University of Pittsburgh in the Spring of 2018. By that time Laurent Taudin, illustrator extraordinaire for many projects of the *Max-Planck-Institut für Wissenschaftsgeschichte* in Berlin, had drawn the figures of the chimps and the bananas that we have been using in talks and lectures since (see Figures 2.1 and 2.2).

After extensive preparatory work by Janas and Janssen in the Fall of 2018, we started writing what would eventually become this book during a visit by Cuffaro to Minnesota in January 2019. In March, Cuffaro presented a preliminary version of parts of Chapters 2, 3 and 6 at the Workshop on Interpreting Quantum Mechanics organized by Giovanni Valente at the *Politecnico di Milano* in Milan. In May, after a test run by Janas in a mathematics colloquium at the University of Minnesota, the three of us then presented parts of these same chapters at the 2019 edition of New Directions in the Foundations of Physics in Viterbo. A question for Janas by Wayne Myrvold in Q&A alerted us to an important gap in one of our key results, which we have since managed to close (see Chapter 3, notes 10 and 11). In June 2019 the three of us met again in Minneapolis. Over the ensuing months we finalized (or so we thought) our manuscript and in October we posted it on the arXiv and on the PhilSci Archive preprint servers. By that time Janas had filled several whiteboards in Tate Hall, housing part of the School of Physics and Astronomy of the University of Minnesota, many times over to go over (preliminary versions of) the results presented in Chapters 2–5 with Janssen and, when in town, Cuffaro. Janas also did the computer programming needed for Section 4.2 and for Figures 2.8 and 2.16. Janssen is responsible for most other figures. Cuffaro handled whatever LaTeX issues we ran into.

Janssen gave two talks on parts of our preprint at the Second Chilean Conference on the Philosophy of Physics organized by Pablo Acuña in Santiago in December 2019, where he had the opportunity to discuss the material in person with Jeff Bub. A slightly revised version of our preprint was then pre-circulated among participants in a symposium on the foundations of quantum mechanics organized by Janssen, Jürgen Jost and Jürgen Renn at the *Max-Planck-Institut für Wissenschaftsgeschichte* in Berlin in January 2020. In this symposium, Cuffaro and Janssen presented parts of what was starting to get referred to as the "Three Mikes Manifesto," a play on the famous *Dreimännerarbeit* (Three men paper) with which Max Born, Werner Heisenberg and Pascual Jordan (1926) consolidated matrix mechanics. Based on feedback from the participants in this symposium (especially Guido Bacciagaluppi, Jürgen Jost, Jürgen Renn and Matthias Schemmel) and from others who had read our preprint, we added further material to Chapter 5 and substantially rewrote Chapters 1 and 6 (especially Section 6.5 on measurement). We also changed the title. The title of our preprint, "Putting probabilities first: How Hilbert space generates and constrains them," would have been fine for a journal article in a special issue devoted to *Bananaworld*. It would have been obvious, for instance, in that context that our topic is quantum mechanics even though the title does not explicitly mention this. Given the use of Hilbert space methods in general probability theory and statistics, however, this would not have been clear for a monograph with that same title. We settled on the new title *Understanding Quantum Raffles*. Raffles of various designs are ubiquitous in this volume. And while we are hardly the first to argue that the basic formalism of quantum mechanics is essentially a new framework for handling probabilities (cf. Chapter 1, notes 16 and 29), we are the first to do so on the basis of a sustained comparison between raffles serving as toy models of local hidden-variable theories and the statistical ensembles characterized by density operators in terms of which John von Neumann (inspired by Richard von Mises) first formulated quantum mechanics (Von Neumann, 1927b). The "quantum raffles" in the title of our book refer to these statistical ensembles introduced by von Neumann.

In the Fall of 2020, after trying out some of the material in Chapter 2 in classes at the University of Minnesota and Washburn High School in Minneapolis, Janssen, assisted by Janas, taught a seminar in the Honors Program of the University of Minnesota under the title of Gilder's (2008) *The Age of Entanglement*, covering—in addition to Gilder's book and the graphic novel *Totally Random* by Tanya and Jeff Bub (2018)—Chapters 1–3 of the manuscript of *Understanding Quantum Raffles*. In response to student feedback, we reorganized some of the material in Chapters 2 and 3.

We are grateful for the questions from the audiences at the various workshops and talks mentioned above as well as for the feedback from students at the University of Minnesota and Washburn High School. In addition, we want to single out a number of individuals not explicitly mentioned so far and thank them for helpful comments and discussion: Jossi Berkovitz, Victor Boantza, Harvey Brown, Časlav Brukner, Adán Cabello, Joe Cain, Cindy Cattell, Radin Dardashti, Michael Dascal, Robert DiSalle, Tony Duncan, Lucas Dunlap, Laura Felline, Sam Fletcher, Mathias Frisch, Chris Fuchs, Louisa Gilder, Sona Ghosh, Peter Gilbertson, Peter Grul, Bill Harper, Stephan Hartmann, Geoffrey Hellman, Leah Henderson, Federico Holik, Luc Janssen, Christian Joas, Molly Kao, David Kaiser, Jim Kakalios, Alex Kamenev, Jed Kaniewski, Marius Krumm, Femke Kuiling, Samo Kutoš, Christoph Lehner, Charles Marcus, Tushar Menon, Eran Moore Rea, Markus Müller, Max Niedermaier, Sergio Pernice, Vincent Pikavet, Serge Rudaz, David Russell, Rob "Ryno" Rynasiewicz, Juha Saatsi, Ryan Samaroo, Chris Smeenk, Rob Spekkens, Jos Uffink, David Wallace and Brian Woodcock. We thank Lindy Divarci, Jürgen Renn and Matteo Valleriani of the *Max-Planck-Institut für Wissenschaftsgeschichte* for their help in turning our manuscript into a book. We thank an anonymous referee who reviewed our book for Springer both for the enthusiastic endorsement and for helpful comments. We thank Lucy Fleet, Prasad Gurunadham and Svetlana Kleiner at Springer for shepherding our manuscript through the production process.

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Lino Lakes, MN, USA *Michael Janas*
Montréal, Québec, Canada *Mike Cuffaro*
Minneapolis, MN, USA *Michel Janssen*

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