Many Worlds, the Cluster-state Quantum Computer, and the Problem of the Preferred Basis*†

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1 Introduction

The source of quantum computational 'speedup'—the ability of a quantum computer to achieve, for some problem domains,[1] a dramatic reduction in processing time over any known classical algorithm—is still a matter of debate. On one popular view (the 'quantum parallelism thesis'[2]), the speedup is due to a quantum computer's ability to simultaneously evaluate (using a single circuit) a function for many different values of its input. Thus one finds, in textbooks on quantum computation, pronouncements such as the following:

[1]An important example is the factoring problem (Shor, 1997).

[2]I am indebted to Duwell (2007) for this label.

[a] qubit can exist in a superposition of states, giving a quantum computer a hidden realm where exponential computations are possible ... This feature allows a quantum computer to do parallel computations using a single circuit—providing a dramatic speedup in many cases (McMahon, 2008, p. 197).

Unlike classical parallelism, where multiple circuits each built to compute $f(x)$ are executed simultaneously, here a *single* $f(x)$ circuit is employed to evaluate the function for multiple values of x simultaneously, by exploiting the ability of a quantum computer to be in superpositions of different states (Nielsen and Chuang, 2000, p. 31).

Among textbook writers, N. David Mermin is, perhaps, the most cautious with respect to the significance of this 'quantum parallelism':

One cannot say that the result of the calculation *is* 2^n evaluations of f, though some practitioners of quantum computation are rather careless about making such a claim. All one can say is that those evaluations characterize the *form* of the state that describes the output of the computation. One knows what the state *is* only if one already knows the numerical values of all those 2^n evaluations of f. Before drawing extravagant practical, or even only metaphysical, conclusions from quantum parallelism, it is essential to remember that when you have a collection of Qbits in a definite but unknown state, *there is no way to find out what that state is* (2007, p. 38).

Mermin's reservations notwithstanding, the quantum parallelism thesis is frequently associated with (and held to provide evidence for) the many worlds explanation of quantum computation, which draws its inspiration from the Everettian interpretation of quantum mechanics. According to the many worlds explanation of quantum computing, when a quantum computer effects a transition such as:

$$\sum_{x=0}^{2^n-1} |x\rangle|0\rangle \rightarrow \sum_{x=0}^{2^n-1} |x\rangle|f(x)\rangle, \tag{1}$$

it literally performs, simultaneously and in different physical worlds, local function evaluations on all of the possible values of x.

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The many worlds explanation of quantum computing is a very attractive explanation of quantum speedup if one accepts the quantum parallelism thesis, for, since the many worlds explanation of quantum computing directly answers the question of *where* this parallel processing occurs (i.e., in distinct physical universes) in a way in which other explanations do not, it is, arguably, the most intuitive explanation of quantum speedup. Indeed, for some, the many worlds explanation of quantum computing is the only possible explanation of quantum speedup. David Deutsch, for instance, writes: "no single-universe theory can explain even the Einstein-Podolsky-Rosen experiment, let alone, say, quantum computation. That is because *any* process (hidden variables, or whatever) that accounts for such phenomena ... contains many autonomous streams of information, each of which describes something resembling the universe as described by classical physics" (2010, p. 542). Deutsch issues a challenge to those who would explain quantum speedup without many worlds: "[t]o those who still cling to a single-universe worldview, I issue this challenge: Explain how Shor's algorithm works" (1997, p. 217).

Recently, the development of an alternative model of quantum computation—the cluster state model—has cast some doubt on these claims. The standard network model (which I will also refer to as the 'circuit' model) and the cluster state model are computationally equivalent in the sense that one can be used to efficiently simulate the other; however, while an explanation of the network model in terms of many worlds seems intuitive and plausible, it has been pointed out by Steane (2003, pp. 474-475), among others, that it is by no means natural to describe cluster state computation in this way.

While Steane is correct, I will argue that the problem that the cluster state model presents to the many worlds explanation of quantum computation runs deeper than this. I will argue that the many worlds explanation of quantum computing is not only unnatural as an explanation of cluster state quantum computing, but that it is, in fact, incompatible with it.[3] I will show how this

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[3]My use of the word 'incompatible' might strike some readers as a touch strong. I do not mean to convey by this any in-principle impossibility, however. Rather, I take it that any worthwhile explanation of a process should provide some useful insight into its workings, and should be motivated by the characteristics of the process, not by predilections for a particular type of explanation on the part of the explainer. My claim here is that, as I will show below, a many worlds explanation of cluster state quantum computing is completely unmotivated and useless even as a heuristic device for describing cluster state quantum computation, and is in this sense incompatible with it.

incompatibility is brought to light through a consideration of the familiar preferred basis problem, for a preferred basis with which to distinguish the worlds inhabited by the cluster state neither emerges naturally as the result of a dynamical process, nor can be chosen a priori in any principled way. In the process I will provide a much needed exposition of cluster state computation to the philosophical community.[4]

In addition, I will argue that the many worlds explanation of quantum computing is inadequate as an explanation of even the standard network model of quantum computation. This is because, first, unlike its close cousin, the neo-Everettian many worlds interpretation of quantum *mechanics*,[5] where the *decoherence* criterion is able to fulfil the role assigned to it, of determining the preferred basis for world decomposition with respect to macro experience,[6] the corresponding criterion for world decomposition in the context of quantum computing cannot fulfil this role except in an ad hoc way. Second: alternative explanations of quantum computation exist which, unlike the many worlds explanation, are compatible with both the network and cluster state model.

The quantum parallelism thesis, and the many worlds explanation of quantum computation that is so often associated with it, are undoubtedly of great heuristic value for the purposes of algorithm analysis and design, at least with regard to the network model. This is a fact which I should not be misunderstood as disputing. What I am disputing is that we should therefore be committed to the claim that these computational worlds are, in

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[4]Apart from very high-level discussions such as those found in Steane (2003), no one, to my knowledge, has yet presented, to the philosophical community, a detailed exposition of cluster state quantum computing.

[5]One should be wary not to treat the 'Everettian' interpretation of quantum mechanics as if it were a unified view. Rather, 'Everettian' more properly describes a family of views (see Barrett 2011 for a list and discussion of these), which includes but is not limited to Hugh Everett's original formulation (Everett, 1957), 'many minds' variants (Albert and Loewer, 1988), and 'many worlds' variants. Belonging to the last named class are DeWitt's (1973 [1971]) original formulation, as well as the, now mainstream, 'neo-Everettian' interpretation with which we will be mostly concerned in this paper. I follow Hewitt-Horsman (who attributes the name to Harvey Brown) in calling 'neo-Everettian' the amalgam of ideas of Zurek (2003 [1991]); Saunders (1995); Butterfield (2002); Vaidman (2008), and especially Wallace (2002, 2003, 2010).

[6]I should not be interpreted here as giving an argument *for* the neo-Everettian interpretation of quantum mechanics. My views on the correct interpretation of quantum mechanics are irrelevant to this discussion. My claim is only that the decoherence basis is prima facie well-suited for the role it plays in the neo-Everettian interpretation.

fact, ontologically real, or that they are indispensable for any explanation of quantum speedup.

My essay will proceed as follows. In order to exhibit the motivations and intuitions for adopting a many worlds view of quantum computation, I will begin, in section 2, with an example of a simple quantum algorithm. In section 3, I will argue that, despite its intuitive appeal, the many worlds view of quantum computation is not licensed by, and in fact is conceptually inferior to, the neo-Everettian version of the many worlds interpretation of quantum mechanics from which it receives its inspiration. In section 4, I will describe the cluster state model of quantum computation and show how the cluster state model and the many worlds explanation are incompatible. In section 5 I will argue, based on the conclusions of sections 3 and 4, that we should reject the many worlds explanation of quantum computation *tout court*.

2 A Simple Quantum Algorithm

The motivation for the view that quantum computation is parallel processing (i.e., the quantum parallelism thesis), is evident when one considers the specification of certain existing quantum algorithms. Consider, for instance, the following simple algorithm for solving Deutsch's problem: the problem to determine whether a boolean function taking one bit as input and producing one bit as output is either constant or balanced.

Such a function is constant if it produces the same output value for each of its inputs. If we consider the functions $f : \{0,1\} \rightarrow \{0,1\}$, the only possible constant functions are $f(x) = 0$ and $f(x) = 1$. A balanced function, on the other hand, is one for which the output of one half of the inputs is the opposite of the output of the other half. For the functions $f : \{0,1\} \rightarrow \{0,1\}$ the only possible balanced functions are the identity and bit-flip functions, which are, respectively:

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x = 1 \end{cases} \qquad\qquad f(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x = 1 \end{cases}$$

Now classically, the only way to determine whether such a function is balanced or constant is to test the function for each possible value of its input (i.e., for 0 and 1). In this case, that amounts to two function invocations. In

a quantum computer, however, we can learn whether the function is balanced or constant by evaluating the function only *once*.

To implement the quantum algorithm using the circuit model,[7] we begin by preparing two qubits[8] (initially assigned the state $|0\rangle$) in the following way. First, we send them each through a NOT (i.e., a Pauli-$\mathbf{X}$) gate, which flips the state of the qubit. We then send them each through a Hadamard gate, which transforms the state of each qubit into a coherent superposition of the classical states, $|0\rangle$ and $|1\rangle$:[9]

$$
\begin{aligned}
(\mathbf{H} \otimes \mathbf{H})(\mathbf{X} \otimes \mathbf{X})(|0\rangle|0\rangle) &= (\mathbf{H} \otimes \mathbf{H})(|1\rangle|1\rangle) \\
&= \tfrac{1}{2}(|0\rangle - |1\rangle)(|0\rangle - |1\rangle) \\
&= \tfrac{1}{2}(|0\rangle|0\rangle - |1\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|1\rangle)
\end{aligned}
$$

We now send the two qubits through a 'black box'—a unitary gate, $\mathbf{U}_f$, representative of the function whose character (of being either constant or balanced) we wish to determine—which will perform the required evaluation. We define $\mathbf{U}_f$ so that it leaves the first qubit alone but XORs the second qubit with the result of evaluating f on the value of the first qubit, i.e.:

$$
\mathbf{U}_f(|x\rangle|y\rangle) =_{df} |x\rangle|y \oplus f(x)\rangle. \tag{2}
$$

Thus we have:

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\begin{aligned}
&\tfrac{1}{2}\mathbf{U}_f(|0\rangle|0\rangle - |1\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|1\rangle) \\
=\ &\tfrac{1}{2}(|0\rangle|0 \oplus f(0)\rangle - |1\rangle|0 \oplus f(1)\rangle - |0\rangle|1 \oplus f(0)\rangle + |1\rangle|1 \oplus f(1)\rangle) \quad (3) \\
=\ &\tfrac{1}{2}(|0\rangle|f(0)\rangle - |1\rangle|f(1)\rangle - |0\rangle|\tilde{f}(0)\rangle + |1\rangle|\tilde{f}(1)\rangle),
\end{aligned}
$$

where $\tilde{f}(x) =_{df} 1 \oplus f(x)$. Note how the action of the unitary transformation gives the appearance of evaluating the function over multiple inputs at once.

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[7]The exposition which follows is similar to Mermin's (2007, p. 44). This is an improved version of the algorithm originally developed by Deutsch (1985, pp. 111-112). Deutsch's original algorithm, unlike the one given here, works probabilistically.

[8]A qubit is the basic unit of quantum information, analogous to a classical bit. It can be physically realized by any two-level quantum mechanical system. Like a bit, it can be "on": $|1\rangle$ or "off": $|0\rangle$, but unlike a bit it can also be in a superposition of these values.

[9]'Logic gates' in the network model of quantum computation are implemented as unitary transformations. The Pauli-$\mathbf{X}$ transformation takes $|0\rangle$ to $|1\rangle$ and vice versa. A Hadamard transformation takes $|0\rangle$ to $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$ and $|1\rangle$ to $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$ and vice-versa.

Now if the function is constant, then $f(0) = f(1)$, $\tilde{f}(0) = \tilde{f}(1)$, and the state can be expressed as

$$\tfrac{1}{2}(|0\rangle - |1\rangle)(|f(0)\rangle - |\tilde{f}(0)\rangle).$$

If the function is balanced, $f(0) \neq f(1)$, but $f(1) = \tilde{f}(0)$ and $\tilde{f}(1) = f(0)$, thus the state can be expressed as:

$$\tfrac{1}{2}(|0\rangle + |1\rangle)(|f(0)\rangle - |\tilde{f}(0)\rangle)$$

If we apply a Hadamard transformation to the first qubit then the state goes to

$$|1\rangle \tfrac{1}{\sqrt{2}}(|f(0)\rangle - |\tilde{f}(0)\rangle), \quad |0\rangle \tfrac{1}{\sqrt{2}}(|f(0)\rangle - |\tilde{f}(0)\rangle) \tag{4}$$

for the constant and balanced scenarios, respectively. All that is left to do, in order to determine whether the function is constant or balanced, is to measure the first qubit in the $\{|0\rangle, |1\rangle\}$ (i.e., the computational) basis.

The algorithm, taken at a glance, is:

$$(\mathbf{H} \otimes \mathbf{I})\mathbf{U}_f(\mathbf{H} \otimes \mathbf{H})(\mathbf{X} \otimes \mathbf{X})(|0\rangle|0\rangle). \tag{5}$$

In this way, we solve the problem with only one invocation of $\mathbf{U}_f$. It indeed appears as though we have performed two steps in one.

3 Neo-Everett and Quantum Computing

Algorithms like Deutsch's and more impressive algorithms like Shor's (which appear to perform many more than two steps in one) provide strong intuitive support for the view that quantum speedup is due to a quantum computer's ability to simultaneously evaluate a function for different values of its input, and from here it is not a large step to the many worlds picture of quantum computation. It is important to note, however, that one's conception of a world, if one elects to take this step, cannot be the one that is licensed by the neo-Everettian many worlds interpretation of quantum mechanics. In superpositions such as the following,

$$\frac{1}{\sqrt{2}}(|\alpha\rangle \otimes |\beta\rangle + |\gamma\rangle \otimes |\delta\rangle),$$

the neo-Everettian interpretation will not, in general, license one to identify each term of this superposition with a distinct world, for such a simplistic

procedure for world-identification will be vulnerable to the so-called preferred basis objection.

The problem is usually formulated in the context of macro-worlds and macro-objects; however we can illustrate the basic idea by means of the following simple example related to quantum computation. The classical value $\uparrow$ can be represented, in the computational basis,[10] by a qubit in the state $|0\rangle$. We can also represent the same qubit from the point of view of the $\{|+\rangle, |-\rangle\}$ basis, however, as[11]

$$\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle).$$

Thus depending on the basis one selects, it will be possible to regard the qubit as either (if we select the computational basis) in the definite state $|0\rangle$, existing in one world only, or (if we select the $\{|+\rangle, |-\rangle\}$ basis), as in a superposition of the two states, $|+\rangle$ and $|-\rangle$, and thus as existing in two distinct worlds. Yet there seems to be no a priori reason why we should elect to choose one basis over the other.

Neo-Everettians (see, for instance, Wallace 2002, 2003) attempt to eliminate the preferred basis problem by appealing to the dynamical process of decoherence (*Cf.* Zurek 2003 [1991]) as a way of distinguishing different worlds from one another in the wave function. Recall that Schrödinger's wave equation governs the evolution of a closed system. In nature, however, there are no closed systems (aside from the entire universe); all systems interact, to some extent, with their environment. When this happens, the terms in the superposition of states representing the system decohere and branch off from one another. From the point of view of an observer in a particular world, this gives the appearance of wave-function collapse—of definiteness emerging from indefiniteness—but unlike actual collapse (i.e., collapse as per von Neumann's projection postulate), decoherence is an approximate phenomenon; thus some small amount of residual interference between worlds always remains. But from the point of view of our experience of macroscopic objects, this is, for all practical purposes, enough to give us the appearance of definiteness within our own world and to distinguish, within the wave-function,

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[10]The computational, or classical, basis for a single qubit is the basis $\{|0\rangle, |1\rangle\}$, which can be used to represent the classical bit states $\{\uparrow, \downarrow\}$, where $|0\rangle = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$, and $|1\rangle = \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right)$. An alternative basis for computation is $\{|+\rangle, |-\rangle\}$, where $|+\rangle = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right)$, and $|-\rangle = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right)$.

[11]Since $|+\rangle = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, $\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) = \frac{1}{2}(|0\rangle + |1\rangle + |0\rangle - |1\rangle) = \frac{1}{2} \cdot 2|0\rangle = |0\rangle$.

macroscopic worlds that evolve essentially independently and maintain their identities over time. Thus, a 'preferred' basis with which one can define different worlds emerges *naturally*: "the basic idea is that dynamical processes cause a preferred basis to emerge rather than having to be specified a priori" (Wallace, 2003, p. 90).

On the neo-Everettian view, we identify patterns which are present in the wave-function and which are more or less stable over time in this way with macroscopic objects such as measurement pointers, cats, and experimenters. But note that not every such pattern is granted ontological status; whether or not we do so depends, not just on the process of decoherence, but also on the theoretical usefulness of including that object in our ontology: "the existence of a pattern as a real thing depends on the usefulness—in particular, the explanatory power and predictive reliability—of theories which admit that pattern in their ontology" (Wallace, 2003, p. 93). Thus, while decoherence is a necessary condition for granting ontological status to a pattern, it is not sufficient; we *also* require that doing so is theoretically useful and fruitful.

Returning to the quantum computer, it should be clear by now that the neo-Everettian interpretation, as described above, cannot provide support for the view that quantum computers simultaneously evaluate functions for different values of their input *in different worlds*, for as we have just seen, *decoherence* determines the basis according to which we distinguish one world from another on the neo-Everettian interpretation. The superpositions characteristic of quantum algorithms, however, are always *coherent* superpositions. Indeed, the maximum length of a quantum computation is directly related to the amount of time that the system remains coherent (Nielsen and Chuang, 2000, p. 278). According to some, in fact, it is coherence and not parallel processing which is the real source of quantum speedup (Fortnow, 2003). Decoherence, in the context of quantum computation, effectively amounts to noise.

It appears, then, that we require a more general criterion for branching than decoherence if we are to accommodate quantum computation to a many worlds picture. Thus, Hewitt-Horsman (2009), who is notable among advocates of the many worlds explanation for presenting a positive argument for the many worlds explanation and not a mere assertion that other explanations are impossible, rejects the idea that decoherence is the only possible criterion for distinguishing worlds. Worlds, for Hewitt-Horsman, are (just as in the neo-Everettian approach), defined as substructures within the wave-function that 'for all practical purposes' are distinguishable and stable over

relevant time scales. With regards to macro experience these relevant time scales are long, and the point of using decoherence as an identifying criterion for distinct worlds, according to Hewitt-Horsman, is that it is useful for identifying stable macro-patterns over such long time scales. But the time scales relevant to quantum computation are generally much shorter: "they may, indeed, be *de facto* instantaneous. However, if they are useful then we are entitled to use them" (Hewitt-Horsman, 2009, p. 876).

In such a situation we may, according to Hewitt-Horsman, consider coherent superpositions as representing distinct worlds for the purposes of characterising quantum computation. "Defining worlds within a coherent state in this way is a simple extension of the FAPP[12] principle ... If our practical purposes allow us to deal with rapidly changing worlds-structures then we may" (Hewitt-Horsman, 2009, p. 876). As for the preferred basis problem, it will not arise. Just as with the neo-Everettian interpretation, in the quantum computer we have a criterion for selecting a basis with which to decompose the wave function; in this case the basis is that in which the different evaluations of the function are made manifest, i.e., the computational basis.

> This fits in well with intuitions that are often expressed about the nature of quantum computations ... There are frequently statements to the effect that it *looks like* there are multiple copies of classical computations happening within the quantum state. If one classical state from a decomposition of the (quantum) input state is chosen as an input, then the computation runs in a certain way. If the quantum input state is used then it looks as if all the classical computations are somehow present in the quantum one. ... the recognition of multiple worlds in a coherent states [*sic.*] seems both to be a natural notion for a quantum information theorist, and also a reasonable notion in any situation where 'relevant' time-scales are short (Hewitt-Horsman, 2009, p. 876).

Certainly it does look as if the computation is composed of many processes executing in parallel, and plausibly it can be of some heuristic value to think of these processes as taking place in many worlds. With this I do not disagree. However, *pace* Hewitt-Horsman, I do not believe this is enough to justify treating these worlds as ontologically real, for unlike the criterion

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[12]FAPP stands for 'for all practical purposes'.

of decoherence with respect to macro experience, Hewitt-Horsman's criterion for distinguishing worlds in the context of quantum computation seems quite ad hoc. Declaring that the preferred basis is the one in which the different function evaluations are made manifest is like declaring that the preferred basis with respect to macro experience is the one in which we can distinguish classical states from one another. But it is, in fact, a rejection of such reasoning that leads to decoherence as a criterion for world-identification in the first place. The decoherence basis, on the neo-Everettian view, is not simply picked from among many possible bases as the one which serves to capture our experience of definiteness at the macro-level. To do so would be to commit the same sin (by neo-Everettian lights) that is committed by other interpretations of quantum mechanics such as Bohmian mechanics or GRW theory. This is the sin of adding extra elements to the formalism of quantum theory in order to preserve classicality at the macroscopic level. For the neo-Everettian, in contrast, decoherence is appealed to as a known physical process that *in fact* gives rise to—and even then only approximately—the appearance of distinct classical worlds (Cf. Wallace, 2010, pp. 55, 63-65). The point of using decoherence as a criterion for distinguishing worlds is not to save the appearance of classicality, but rather to *explain* why we experience the world classically, in this case[13] by appealing to a physical process that gives rise to our experience. The choice of the computational basis as the basis within which different worlds are to be distinguished, however, fulfils no such explanatory role. It does not serve to explain the appearance of parallel classical computation. It only declares, based on a particular privileged description of the computation, that parallel computation is occurring in many worlds.[14]

An advocate of the many worlds explanation might make the following rejoinder: the computational process, considered as a whole, is just as empirically well-established as the decoherence process is (we know that a compu-

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[13]Note that I am not taking sides here in the debate over whether it is necessary to appeal to causes in such explanations.

[14]I should mention that Wallace, who I am taking as representative of the neo-Everettian interpretation of quantum mechanics, does seem to cautiously endorse a many worlds explanation for *some* quantum algorithms: "There is no particular reason to assume that *all* or even *most* interesting quantum algorithms operate by any sort of 'quantum parallelism' ... But Shor's algorithm, at least, does seem to operate in this way" (Wallace, 2010, p. 70, n. 17). Wallace has also made similar remarks in informal correspondance. But whatever Wallace's views on quantum computation are, they are obviously separable from his views on world decomposition for macro-phenomena.

tation has taken place since we have the result). And just as the decoherence process gives rise to parallel autonomously evolving decoherent worlds which are (approximately) diagonal in the decoherence basis, the computational process gives rise to parallel autonomously evolving computational worlds which are diagonal in the computational basis. Thus the computational process gives rise to and therefore explains the computational worlds that make up the computation just as well as the decoherence process explains the decoherent worlds that make up classical experience.

This response is problematic, however, for it is the computation itself, in particular what distinguishes it from classical computation, that we are seeking an explanation for. The many worlds explanation of quantum computation promises to explain quantum computation in terms of many worlds, but on this response it appears that we need to appeal to the computation in order to explain these many worlds in the first place. This seems circular, and even if the case can be made that it is not, the response fails to consider that, as the Mermin quote with which I began this paper makes clear, appearances can be misleading: we must be very cautious when describing the quantum state characterising a computation. In particular, we must be cautious when inferring from the form of the state that describes the computation to the content of that state. For instance, as Steane (2003, p. 473) has pointed out, according to the Gottesman-Knill theorem, an important class of quantum gates—the so-called Clifford-group gates, which include the Hadamard, Pauli, and CNOT gates—can be simulated in polynomial time by a classical probabilistic computer (Nielsen and Chuang, 2000, p. 464). This is interesting, since several quantum algorithms utilise gates exclusively from this class. Thus the appearance of quantum parallelism in these cases may be deceiving.

Even if true, the quantum parallelism thesis need not entail the existence of autonomous local parallel computational processes. Duwell (2007, p. 1008), for instance, illustrates this by showing how the phase relations between the terms in a system's wave function are crucially important for an evaluation of its computational efficiency. Phase relations between terms in a system's wave function, however, are global properties of the system. Thus we cannot view the computation as consisting exclusively of local parallel computations (within multiple worlds or not). But if we cannot do so, then there is no sense in which quantum parallelism uniquely supports the many worlds explanation over other explanations.

Everettian varieties such as the neo-Everettian interpretation of quan-

tum mechanics and the many worlds explanation of quantum computing take the branching process *seriously*: they claim ontological significance for the 'worlds' that arise from this process. They are thus required to confront the preferred basis problem, for they must determine a criterion for branching. While the neo-Everettian interpretation of quantum *mechanics* does this admirably well, the many worlds explanation of quantum *computing*, I have argued, does not.

Before concluding this section, I should note that not all Everettian varieties do take branching seriously (in fact, this may have been true of Everett's own view. See Barrett 2011 for a discussion). Such views are not confronted with the preferred basis problem and are thus immune to the objections above. However, since branching is not a real physical process on such views, it is analytic that they can provide no physical explanation for the quantum computational process in terms of branching computational worlds. As an illustration, an Everettian might insist[15] that the way in which one chooses to express the state of a system has no particular significance. On such an interpretation, one should not view the universe as having any one particular branching structure. Rather, the essential point is that in any process such as quantum computation, the fact is that the Schrödinger evolution of *all* quantum superpositions has been realised. On such a view, *however* one chooses to decompose the state of a system, the resulting superposition must be viewed as real. Thus the superposition of the quantum computational process, as expressed in the computational basis, is realised, just as is the superposition as expressed in some other basis.

On this interpretation, however, it cannot be the case that multiple local parallel computational processes in many worlds are *the explanation* for quantum speedup; for any decomposition of the state of the computer in any given basis can provide the ground for an equally legitimate 'explanation' of the computer's operation. Rather, we should say that any such decomposition constitutes, for one who finds Everettian language appealing, a legitimate *description* of the process. I do not wish to be misunderstood as attempting to deny to those who find Everettian language appealing the possibility of, when appropriate, describing the operation of the quantum computer in this way. But again, this does not constitute an *explanation*.

In any case, the questionable nature of the inference from the heuristic

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[15]I am indebted to an anonymous reviewer at Studies in History and Philosophy of Modern Physics for pointing this out.

value of the notion of computational worlds to the ascription of ontological reality to these worlds is one good reason to, at the very least, be suspicious of the many worlds explanation of quantum computing. But let us, for the sake of argument, grant the inference. Let us focus, instead, on the antecedent clause of the conditional; i.e., on whether it really is true that the many worlds description of quantum computation is the most useful one available. In the next section I will examine the recently developed cluster state model of quantum computation. I will argue that a description of the cluster state model in terms of many worlds is, not only unnatural, but that such a description is incompatible with the cluster state model. I will then argue that this undermines the usefulness of the many worlds description of quantum computation, not just in the cluster state model, but in general.

4   Cluster State Quantum Computing

On the cluster state model (Raussendorf and Briegel, 2002; Raussendorf et al., 2003; Nielsen, 2005) of quantum computation, computation proceeds by way of a series of single qubit measurements on a highly entangled multi-qubit state known as the cluster state.[16] The cluster-state quantum computer ($QC_{\mathcal{C}}$) is a universal quantum computer; it can efficiently simulate any algorithm developed within the network model. In fact it is computationally equivalent to the network model in the sense that each model may be used to simulate the operation of the other. Each qubit in the cluster has a reduced density operator of $\frac{1}{2}\mathbf{I}$, and thus individual qubit measurement outcomes are completely random. It is nevertheless possible to process information on the cluster state quantum computer due to the fact that strict correlations exist between measurement outcomes. These correlations are progressively destroyed as the computation runs its course.[17]

Since most readers are familiar with the network model, it will be easiest to illustrate the operation of the $QC_{\mathcal{C}}$ by exhibiting the way one goes about simulating a network-based algorithm with the $QC_{\mathcal{C}}$. In the network model, single-qubit gates can, in general, be thought of as rotations of the Bloch sphere. For example, the Pauli X, Y, and Z gates can be thought of as rotations of the Bloch sphere through π radians about the x, y, and z axes,

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[16]For this reason the model has also been given the name 'measurement based computation'.

[17]This gives rise to a third name for this model: 'one-way computation'.

respectively. Now, it is possible to simulate an *arbitrary* rotation of the Bloch sphere with the $QC_{\mathcal{C}}$ by using a chain of 5 qubits as follows (*Cf.* Raussendorf and Briegel 2002, pp. 446-447, Raussendorf et al. 2003, p. 5). First, we consider the Euler representation of an arbitrary rotation.[18] This is

$$U_{Rot}[\xi, \eta, \zeta] = U_x[\zeta]U_z[\eta]U_x[\xi], \tag{6}$$

where the rotations about the x and z axes are given by

$$U_x[\alpha] = \exp\left(-i\alpha\frac{\sigma_x}{2}\right), \tag{7}$$

$$U_z[\alpha] = \exp\left(-i\alpha\frac{\sigma_z}{2}\right). \tag{8}$$

The first qubit in the chain is called the input qubit; it will contain the state that we wish to rotate. It is thus prepared in the state $|\psi_{in}\rangle$, while the other four qubits in the chain are prepared in the $|+\rangle$ state. After applying an entanglement-generating unitary transformation to the qubits,[19] the first four qubits are measured one by one in the following way. We begin by measuring qubit 1 in basis $\mathcal{B}_1(0)$, where 0 is the measurement angle, ϕ_j, and the basis is calculated as

$$\mathcal{B}_j(\phi_j) = \left\{ \frac{|0\rangle_j + e^{i\phi_j}|1\rangle_j}{\sqrt{2}}, \frac{|0\rangle_j - e^{i\phi_j}|1\rangle_j}{\sqrt{2}} \right\}. \tag{9}$$

The result of this measurement is denoted s_1, where $s_j \in \{0, 1\}$ represents the result of measuring the jth qubit.

We now use s_1 to calculate the measurement basis for qubit 2, which is $\mathcal{B}_2(-\xi(-1)^{s_1})$. Qubit 2 is then measured in this basis and the result recorded

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[18]The Euler representation is a way to represent the general rotation of a body in three dimensions. The procedure to achieve such a general rotation consists of three steps: a rotation of the body about one of its coordinate axes, followed by a rotation about a coordinate axis different from the first, and then a rotation about a coordinate axis different from the second. We represent rotations by Rotation operators, and matrix multiplication is used to represent combinations of rotations. For example, a rotation of α about $\hat{z}$ followed by a rotation of β about $\hat{y}$ followed by a rotation of γ about $\hat{x}$ is represented by $R_x(\gamma)R_y(\beta)R_z(\alpha)$. The analogue of the rotation operator in a complex state space is the unitary operator.

[19]The procedure for generating entanglement is described in (Raussendorf et al., 2003, pp. 3-4).

in s_2, which is then used to determine the measurement basis for qubit 3: $\mathcal{B}_3(-\eta(-1)^{s_2})$. We then use both s_1 and s_3 to determine the basis to use for the measurement of qubit 4: $\mathcal{B}_4(-\zeta(-1)^{s_1+s_3})$. At the end of this process, the output of the 'gate' is contained in qubit 5 (i.e., qubit 5 is in a state that is equivalent to what would have resulted if we had applied an actual rotation to $|\psi_{in}\rangle$), which we then read off in the computational basis.[20]

Similarly, it is possible to implement more specific 1-qubit rotations such as the Hadamard, $\pi/2$-phase, X,Y, and Z gates. 2-qubit gates, such as the CNOT gate, can be implemented using similar techniques (Raussendorf et al., 2003, pp. 4-5) and we can combine all of these gates together in order to simulate an arbitrary network.

To illustrate the general operation of the cluster state computer, imagine, once again, that we are simulating a network-based quantum algorithm. In each individual gate simulation there will be, on the one hand, those qubits whose measurement depends on the outcomes of one or more previous measurements for the determination of their basis, and on the other hand, those that do not. We divide these qubits into disjoint subsets, Q_t, of the cluster $\mathcal{C}$, as follows. All qubits, regardless of which gate they belong to, which do not require a previous measurement for the determination of their basis are added to the class Q_0. We then add to Q_1 all qubits which depend solely on the results of measuring qubits in Q_0 for the determination of their basis. Q_2 comprises, in turn, all qubits which depend on the results of measuring qubits in $Q_0 \cup Q_1$ for the determination of their basis. And so on until we reach $Q_{t_{max}}$.

We then begin by measuring the qubits in the set Q_0. We use the outcomes of these measurements to determine the measurement bases for the qubits to be measured in Q_1. Once these are measured, the outcomes of Q_0 and Q_1 together are used to determine the measurement bases for Q_2. The process continues in this fashion until all the required qubits have been measured (Raussendorf et al., 2003, p. 19). Note that the temporal ordering of measurements on the cluster state will, in general, not depend on what role—input, output, etc.—qubits have with respect to the network model. In fact, those qubits that play the role of gates' 'output registers' will typically be among the first to be measured (Raussendorf et al., 2003,

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[20]I have simplified this procedure slightly. The gate simulation actually realizes, not exactly U_{Rot}, but $U'_{Rot}[\xi,\eta,\zeta] = U_{\Sigma,Rot}U_{Rot}[\xi,\eta,\zeta]$, where $U_{\Sigma,Rot} = \sigma_x^{s_2+s_4}\sigma_z^{s_1+s_3}$ is called the random byproduct operator and is corrected for at the end of the computation (Raussendorf et al., 2003, p. 5).

p. 19). In general, the temporal ordering of measurements on a $QC_\mathcal{C}$ that has been designed to simulate a network does not mirror the temporal ordering the gates would have had if they had been implemented as a network (Raussendorf and Briegel, 2002, p. 444).

At this point we must ask ourselves whether it is possible to describe the cluster state model using a many worlds ontology. At first glance there does not seem to be anything barring such a description in principle. We might view each of the qubits as existing simultaneously in multiple worlds, for example, while the computation is being performed. But even if this were possible, it is difficult to see what would be gained by such a description, for this is neither a natural view of what is happening, nor a particularly useful one: in the network model it seems natural to conceive of a unitary gate as effecting a parallel computation by means of a transformation such as that in equation (1). But such a 'step' is missing in the cluster state model. There is nothing corresponding to such a unitary transformation. At best we have a simulation of such a gate; however, it is a simulation that bears no resemblance, in terms of its physical realisation, to the corresponding network circuit. In addition, the temporal ordering of computation in the cluster state has little, if anything, to do with the temporal ordering present in the simulated network. Thus there is nothing corresponding to simultaneous function evaluation in the cluster state, for on the cluster state model gates are only conceptual entities that one may utilise for algorithm design. When it comes to implementation, the logical division of the cluster into distinct gates is completely irrelevant. Indeed, in order to characterise the cluster state model it is not necessary to begin with the logical layout of the network model at all, for the cluster state model is, arguably, more effectively characterised by a graph than by a network (Raussendorf et al., 2003, p. 20).

Far from being a natural and intuitive picture of cluster state computation, it seems, rather, that one must work *against* one's intuition to view the cluster state model as a model of parallel computation in many worlds, and it is hard to see how such a description can be useful. Considerations such as these prompt Steane to write: "[t]he evolution of the cluster state computer is not readily or appropriately described as a set of exponentially many computations going on at once. It is readily described as a sequence of measurements whose outcomes exhibit correlations generated by entanglement" (2003, p. 474). Hewitt-Horsman, also, reluctantly rejects the view that cluster state computation need involve an appeal to many worlds

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(Hewitt-Horsman, 2009, pp. 896-897); however, as we have seen, she still defends the legitimacy and usefulness of describing *network* based computation in terms of many worlds and of treating these worlds as ontologically real (Hewitt-Horsman, 2009, pp. 890-896).

But the main problem, for one who wishes to defend a many worlds description of the operation of the cluster state computer, is not that such a description is neither natural nor useful. The problem is deeper than this, for it appears that it is for all practical purposes impossible to specify a preferred basis in which to distinguish the worlds in which parallel computations take place in the context of the cluster state computer. Recall that, in general, measurements in the cluster state model are *adaptive*: the basis for each measurement will change throughout the computation and will differ from one qubit to the next. During each time step of the computation, the (random) results of the measurements performed in that step will determine the measurement bases used to measure the qubits in subsequent steps. But this random determination of measurement bases means that there is no principled way to select a preferred basis a priori (and even if we did, few qubits would actually be measured in that basis), and we certainly cannot assert that there is any sense in which a preferred basis 'emerges' from this process. Thus there is no way in which to characterise the cluster state computer as performing its computations in many worlds, for there is no way, in the context of the cluster state computer, to even define these worlds for the purposes of describing the computation as a whole.

As a possible rejoinder, one might assert that the cluster state model merely obscures the fact that the computation takes place in many worlds, and that this would be revealed upon closer analysis by, for instance, considering how one might go about simulating a cluster-state computation with circuits. In fact it is possible to simulate a cluster state using classically controlled gates. Classically controlled gates are gates whose operation is dependent on classical bit values (these are typically the results of measurements). To avoid the problem of the continually changing basis, one might take the additional step of deferring all measurements to the end of the process. According to the principle of deferred measurement (Nielsen and Chuang, 2000, p. 186), this is always possible.

Such a simulation would require many more qubits and at least one more two-qubit operation for each single qubit operation in the cluster, however. In principle, there will be no bound to either the additional memory or to the number of additional two-qubit gates required to realise the simulation

(de Beaudrap, 2009, p. 2). Practical methods, therefore, for simulating the cluster state with circuits allow measurement gates to be a part of the computational process (Childs et al., 2005; de Beaudrap, 2009). They decompose the cluster state into a series of classically controlled change of basis gates followed by measurement gates in the standard basis. Thus this will not solve the problem for the many worlds theorist.

But perhaps some day an ingenious theorist will find a way to simulate cluster state computation in some other model without the use of adaptive measurements or classically controlled change of basis gates. What should we say then? Even in this case I think it would be misleading to speak of the cluster state model as obscuring the fact that many worlds are responsible for the speedup it evinces. Recall that, for those who adhere to the many worlds explanation of quantum computation, the motivation for describing computation as literally happening in many worlds is that it is useful for algorithm analysis and design to believe that these worlds are real. This motivation is absent in the cluster state model irrespective of whether it can be simulated in some other model. Moreover, irrespective of whether it can be simulated in some other model, the cluster state model will, in virtue of its unique characteristics, surely lead to new ways of thinking about quantum computation that would not have occurred to a theorist working only with the network model. To dogmatically hold on to the view that, in actuality, many worlds are, at root, responsible for the speedup evinced in the cluster state model will at best be useless, for, as we have seen, it will not help our theorist to design algorithms for the cluster state. At worst it will be positively detrimental if dogmatically holding on to this view prevents our theorist from discovering the possibilities that are inherent in the cluster state model.

5   The Legitimacy of the Many Worlds Explanation for the Network Model

We saw, in section 3, that the many worlds explanation of quantum computing cannot avail itself of many of the arguments in support of the many worlds interpretation of quantum mechanics which appeal to decoherence as a criterion for distinguishing worlds in order to circumvent the preferred basis objection. Further, we saw that while the decoherence basis is able to

fulfil the role assigned to it, in the many worlds interpretation of quantum mechanics, of determining the preferred basis for world decomposition with respect to macro experience, the corresponding criterion for world decomposition appealed to by those who defend the many worlds explanation of quantum computing cannot fulfil this role except in an ad hoc way. Thus we have one reason to reject many worlds as an explanation of the network model of quantum computation. Let us put this consideration to one side.

We have just seen, in section 4, that the cluster state model of quantum computation is incompatible with a many worlds explanation of it. In spite of this, one might still wish to maintain the view that network-based computation, at least, is computation in many worlds. There is nothing wrong in principle with such a stance. What makes this view problematic, however, is the fact that the cluster-state model is computationally equivalent to the network model. One must therefore be committed to the view that an algorithm, when run on quantum circuits, performs its computation in many worlds; while a simulation of the same algorithm, run on a cluster-state computer, does not. Moreover, this is in spite of the fact that there may be no difference in the way in which individual qubits are physically realised in each computer.

As unfortunate as such a situation would be, it would be forced on us if there were no other potential unifying explanations of the source of quantum speedup available. Fortunately, however, there do exist potential explanations for quantum speedup in the network model which, unlike the many worlds explanation, are compatible with the cluster state model. For instance, Steane's choice, entanglement, is one candidate for the source of quantum speedup that has been receiving much attention in the literature. Entanglement has been exploited, in one way or another, in every quantum algorithm developed thus far that has exhibited exponential speedup over classical computation, and entanglement has been proven to be a necessary condition for quantum speedup when using pure states (Jozsa and Linden, 2003).[21] On this view, quantum computers are faster than classical computers because they perform *fewer*, not more, computations. By means of entanglement, quantum computers make it possible to manipulate the correlations

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[21]It has recently been shown, however, that one may achieve, using *mixed* states, a modest speedup over classical computation even when there is no entanglement present, and there are indications that it is possible to achieve exponential speedup in this way as well. It still appears that quantum correlations of some kind are required for quantum speedup, which some have named 'quantum discord'. See Vedral 2010 for a discussion.

present between the logical elements of a computation without representing these elements themselves (Steane, 2003, p. 477). Jeffrey Bub's (2006; 2010) view is somewhat similar. On this view, again, far from computing all of the values of a function simultaneously, quantum computers are faster because they avoid the calculation of any values of the function whatsoever, this time by exploiting the difference between classical and quantum logic. On Lance Fortnow's abstract matrix formulation of the computational complexity class associated with quantum computing, interference is identified as the key difference between it and the corresponding complexity class for classical probabilistic computation.[22] Fortnow develops an abstract mathematical framework for representing these computational complexity classes. In Fortnow's framework, both classes of computation are represented by transition matrices which determine the possible transitions between the configurations of a nondeterministic Turing machine. This framework shows, according to Fortnow, that the fundamental difference between quantum and classical computation is interference: in the quantum case, matrix entries can be negative, signifying a quantum computer's ability to realise good computational paths with higher probability by having the bad computational paths cancel each other out (Fortnow, 2003, p. 606).

Unlike the many worlds explanation, these explanations of the source of quantum speedup do not rely on the particular characteristics of the network model and seem straightforwardly compatible with cluster state computation. But the fact that the many worlds explanation of quantum speedup is not compatible with the cluster state model, while these other explanations of quantum speedup are, is a reason to question its usefulness as a description of network-based quantum computation, and thus one more reason to reject it as an explanation of quantum speedup *tout court*.

6   Conclusion

I hope to have convinced the reader that, whatever the merits of the neo-Everettian interpretation of quantum mechanics are, the many worlds explanation of quantum computing is inadequate as a description of either the network or the cluster state model of quantum computation. We saw above how it depends on a suspect extension of the methodology of the neo-

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[22]These are: These are **BPP** (bounded-error probabilistic polynomial time) and **BQP** (bounded error quantum polynomial time). Cf. Nielsen and Chuang (2000).

Everettian approach to quantum mechanics, and we saw how, unlike other explanations of quantum computing, it is unable to describe the cluster state model of computation. I hope that the reader agrees that these are convincing reasons to reject the many worlds explanation of quantum computing.

I do not want to argue that the many worlds explanation of quantum computation, particularly in regards to the network model, has no heuristic value. It undoubtedly does, and thinking in this manner has assuredly led to the development of some important quantum algorithms. Nevertheless we should take talk of many computational worlds with a grain of salt. Indeed, taking literally the many worlds view of quantum computation may be positively detrimental if it prevents us from constructing models of quantum computation, such as the cluster state model, in the future.

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