

Kant on Pure Intuition, Inner Sense, Self-Affection, and Time, and Gödel on Kant

Michael E. Cuffaro^a

^aMunich Center for Mathematical Philosophy, Ludwig-Maximilians-Universität München

Abstract

Kant's doctrine of inner sense, i.e., our sense, literally, of ourselves, and what according to Kant is its characteristic form: time, bears a critically important relationship to Kant's philosophy of (exact) natural science, broadly understood, and in this chapter I will argue, in addition, that it helps to focus the objections that thinkers, such as Gödel, had to what they understood to be the Kantian programme. I will explain how, according to Kant, space and time are the pure forms of outer and inner sense, respectively, which respectively correspond to our capacity to represent objects as being outside ourselves, and our capacity to represent anything at all. I will explain that attending to the synthesis of what is given to us in outer sensibility invariably produces, for Kant, an effect upon our inner sense, and a corresponding determinate intuition that we relate, in time, to the other determinate intuitions that we have of ourselves and of our own inner state. In this way we perceive ourselves through time, where perception, importantly, involves a kind of empirical consciousness of one's own inner state in relation to some outer appearance. I will then explain that Kant's transcendental schemata correspond to transcendental time determinations, and how the transcendental schemata underlie Kant's synthetic a priori principles, which in turn provide the ground for his metaphysical foundations of natural science. Finally, I relate the discussion to Gödel's ideas on mathematical intuition and to Gödel's objections to and his understanding of Kant's programme. I will explain that intuition, for both Kant and Gödel, is defined in terms of a rule-governed activity, and that Gödel was right to say that there is an analogy between sense perception and mathematical intuition, if the former is understood (as it probably was for Gödel) in the Kantian sense. For Kant, however, intuition is necessarily sensible insofar as the images produced by the power of our imagination are of necessity sensible images, even in the case where what is being exhibited is a temporal interval, and even though time itself cannot be directly perceived. There is no analogous requirement of visualisability, in the case of mathematical intuition, for Gödel.

1 Introduction

In his various remarks on what he called mathematical intuition, the philosopher-mathematician Kurt Gödel (1906–1978), to the bewilderment, or at any rate the dissatisfaction of many of his later commentators,¹ was wont to suggest that the relation

¹For further discussion, see Yourgrau (2024).

between it and the objects of mathematics is analogous to the connection to material objects that is provided to us by perception.

[D]espite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e. in mathematical intuition, than in sense perception. (Gödel, 1983, p. 483–484; quoted in Martin, 2005, p. 207).

Gödel was also wont to emphasise the extent to which he was influenced by (even when he did not agree with) the ideas of Immanuel Kant (1724–1804). In response, for instance, to a question sent to him as part of a questionnaire by one of the editors of his collected works, Burke D. Grandjean, in 1974: “How much importance, if any, do you attribute to each of the scholars in the above list [Ludwig Boltzmann, Jan Brouwer, Paul Finsler, Kant, Karl Kraus, Fritz Mauthner, Jules Richard, and Ludwig Wittgenstein], in the development of your interests?,” Gödel replied: “Only Kant was imp[ortant].” (Wang, 1987, p. 17).² Gödel's engagements with Kant's ideas in mathematics and physics continued throughout his life (Wang, 1987, pp. 27–28, 182–185, 205–208). “But now, if the misunderstood Kant has already led to so much that is interesting in philosophy,” he wrote in the closing sentence of an unpublished manuscript on the foundations of mathematics and philosophy in (roughly) 1961, “... how much more can we expect from Kant understood correctly?” (Gödel, 1995, p. 387).

We will come back to Gödel at the end of the chapter, where I will remark upon what, in light of the following exposition, are a number of interesting historical and conceptual aspects of the larger questions raised by the block quotation above. But our main topic from here on will be Kant, his doctrine of inner sense, i.e., our sense, literally, of ourselves, and what according to Kant is its characteristic form: time. Kant's doctrine is widely considered to be one of the most obscure and difficult parts of his philosophy. In comparison with what Kant calls outer sense, and its characteristic form: space, it has received relatively little attention within Kant scholarship.³ This is unfortunate, since as we will see, it bears a critically important relationship to Kant's philosophy of (exact) natural science, broadly understood, and it helps to focus the objections that those, including Gödel, had to what they understood to be the Kantian programme.

The rest of the chapter will proceed as follows. In Section 2 I will explain that, according to Kant, space and time are the pure forms of outer and inner sense, respectively, which respectively correspond to our capacity to represent objects as being outside ourselves, and our capacity to represent anything at all, the latter being the source of the intuition of our self and of our own inner state. We will see that only space is directly intuitable, but it functions as an intermediary through which we are

²The partially filled-out questionnaire and two variants of a written response were never sent back to Grandjean, but the two men corresponded multiple times, including by telephone, over the course of 1974 and 1975. Gödel elaborated slightly on his relationship to Kant in both variations of his written response: (1) “As to [your question] I would like to say that only Kant had some infl[uence] on my phil[osophical] thinking in gen[eral] & that I got acq[uaunted] with him about 1922”; (2) “None of the scholars mentioned ... in your questionnaire, except perhaps Kant had any direct influence on the development of my interests.” (Wang, 1987, pp. 19–20).

³For some relatively recent literature on Kant's doctrine of inner sense and self-affection, see, for instance, Dyck (2006, 2016), Indregard (2017), and Valaris (2008).

able to indirectly intuit the passage of time. In Section 3 I will explain how the pure forms of sense can also, according to Kant, be thought of as pure intuition, and how Kant distinguishes pure intuition from the images that are exhibited through it by our power of imagination. In Section 4 we will narrow our focus to inner sense. I will explain that, for Kant, we intuit ourselves, through inner sense, only as we appear to ourselves, not as we are in a transcendently realist sense, and I will discuss an objection by Henry Allison. Then, with the aim of further clarifying Kant's doctrine of inner sense, in Section 5, we will turn to Kant's transcendental deduction of the categories of the understanding, and flesh out what it means to appear to oneself, i.e., Kant's doctrine of self-affection. We will see that any combination of the manifold of intuition by our general faculty of understanding, i.e., our power to conceptualise, involves what Kant calls the synthesis of apprehension of what is given in sensibility (i.e., given without spontaneity), and we will see how, on Kant's view, attending to this act of synthesis produces an effect upon our inner sense, and a corresponding determinate intuition that is then related, in time, to the other determinate intuitions that we have of ourselves and of our own inner state through this activity. In this way we perceive ourselves through time, where perception, importantly, involves a kind of empirical consciousness of one's own inner state in relation to some (outer) appearance. In Section 6 we will turn to Kant's paralogisms of pure reason, where we will see that, for Kant, the object of inner sense, unlike the object of outer sense, is immediately perceived. We will then see that what Kant called, in his chapter on the schematism of the pure concepts of the understanding, the transcendental schemata, correspond to pure intuition; in particular: to transcendental time determinations in accordance with the categories, and how the transcendental schemata underlie Kant's philosophy of (exact) natural science, broadly construed. Then, in Section 7, we will discuss the broader influence of Kant's ideas. I will explain how Hermann von Helmholtz, Jakob Friedrich Fries, and Ernst Cassirer all took themselves to have improved upon Kant's understanding of the nature of intuition in various ways. Finally, I will attempt to connect this all back to Gödel, and to the proposition with which we began this introduction.

2 Inner and outer sense and their pure forms

Our cognition of an object considered as such, according to Kant, has two distinct but equally important components. On the one hand, there is the *concept* of the object. On the other hand, there is the *intuition* of it that results from its effects upon our senses (A19/B33).⁴ These represent, so to speak, what we take ourselves to know about the object, and how, ultimately, we take ourselves to know it. The object of intuition, when it is undetermined (*unbestimmte*), is called *appearance* (A20/B34). A dimly lit shape that one can barely make out against the far wall of a dark room, for instance, is the appearance of something indeterminate. But once (after, say, turning the lights on) we determine the manifold of our intuition in accordance with a particular concept, we then understand the object to be a chair, a desk, a dog, etc. (see also Harper, 1984).

In general one can relate the content, or matter, of an appearance both to the object cognised as well as to the cognising subject. Kant's critical philosophy gets off the

⁴References to the *Critique of Pure Reason* (*Kritik der reinen Vernunft*) are to Guyer and Wood's English translation (1998) unless otherwise specified. The page numbers are as in the standard German edition of the Critique, where "A" refers to Kant's first (1781) edition and "B" to his second (1787) edition.

ground with a number of positive claims about the latter. To begin with: *outer sense*, according to Kant, is the property of our mind that enables us to represent objects as being outside ourselves. The formal condition for doing so is to represent them as being in *space*; i.e., space is the *pure form* of outer sense (A22/B37) and one of the two pure forms according to which appearances are related by us in general, where by a *pure form* we mean one that is valid irrespective both of whatever particular sensation occasions a given appearance, and of whatever concept is being used to determine it (A22/B36). Next: *inner sense* is the source of the intuition of our self and of our inner state, and is what enables us to represent anything at all. Its pure form, i.e., the form by which our various mental representations are related to one another within our (evolving) inner state, is *time* (A33/B49). But unlike space, which we are capable of directly intuiting, time is only indirectly intuitable via the intermediary of space. As Kant explains:

[We] represent the temporal sequence through a line progressing to infinity, in which the manifold constitutes a series that is of only one dimension, and infer from the properties of this line to all the properties of time, with the sole difference that the parts of the former are simultaneous but those of the latter always exist successively. From this it is also apparent that the representation of time is itself an intuition, since all its relations can be expressed in outer intuition. (A33/B50).

In other words, it is always possible to express temporal relations via a spatial analogy such that the temporal parts of a thing are put into correspondence with the (spatial) parts of a line. Moreover, according to Kant, the fact that we can do this at all is a justification for calling time an intuition. Kant's argument here seems to be that since space is an intuition, in the specific sense that as an infinite, given, magnitude, it can never be fully captured by any particular (discursive) concept (A25/B39–40),⁵ the fact that parts of time can be put into correspondence with parts of space means that time is intuitive in the very same sense. Our intuition of time is only indirect, however, in the sense that although representing time as a line yields (outer) intuition, this is not the intuition of time, for unlike the temporal parts of the subject of a particular history (such as myself), a line drawn on paper (or on anything else) does not have a temporal beginning or ending but exists for us all at once (see also Allison, 1983, p. 256).

3 Space and time as pure intuition

In the last section we saw that space and time are the pure forms of outer and inner sense, respectively. In this section we will see that we may also think of them as *pure intuition* in the following sense.

So if I separate from the representation of a body that which the understanding thinks about it, such as substance, force, divisibility, etc., as well as that which belongs to sensation, such as impenetrability, hardness, color, etc., something from this empirical intuition is still left for me, namely extension and form. These belong to the pure intuition, which occurs *a priori*,

⁵Kant sometimes refers to space and time as concepts in a more general (i.e., non-discursive) sense (A22/B37, A156/B195).

even without an actual object of the senses or sensation, as a mere form of sensibility in the mind (A20–21/B35).

Let us take a moment to unpack this. Kant's point here is not that there are things, which we call pure intuitions, that can somehow come to exist in our minds altogether apart from the contributions of the understanding and sensibility. Rather, his point is that even if we accept that there can be no relations of time or space without anything (i.e., any representations of things) for them to relate, it is still the case that we are able to consider, in abstract terms, what is left once one removes, in thought, whatever particular is actually being sensed and the particular way it is being understood in the context of a given experience. Moreover, as we will see shortly, via the images produced by our power of imagination, we can reason about the properties of such pure intuition even without anything actually being given to us in sensation. But regardless of whether we reason about the general properties of pure intuition on paper or in our imagination, it is never the case that the pure intuition of space and time can somehow be presented to the mind without any accompanying content of some kind.

[I]ndeed, [space and time's] representation is a mere schema, which is always related to the reproductive imagination that calls forth the objects of experience, without which they would have no significance; and thus it is with all concepts without distinction (A156/B195).

Thus, even if one is asked, for example, to imagine a square simply in the abstract, one necessarily needs, according to Kant, to imagine some thing that encloses this space (typically, an image of four equal-length lines drawn at right angles in the familiar manner). However, it is important that what corresponds to pure intuition in this example is not the imagined four lines, even if it is the case that some such image always accompanies our intuition of a square. Indeed, if somehow the image produced by my imagination could be recorded, then replayed and subjected to scrutiny, I might find that the lines that I had imagined to be enclosing what I had intended to be a square actually vary, slightly, in length or in thickness, or that the angles between them are only approximately right. None of this has any bearing on the content of the pure intuition underlying the image, however, for it is precisely through the pure intuition that I call forth upon drawing a grid over the recorded image, of my previously imagined square, on (for instance) a computer screen, that I am able to determine that it was not quite a square after all.

Consider, as a further example, the constellation Orion (see Figure 1). The characteristic shape of the constellation that we imagine as we look up at it in the sky is, of course, not a property of the stars themselves. Moreover, it arguably misleadingly characterises the actual spatial relations between these stars insofar as, among other things, it emphasises some spatial relations, which just so happen to be evocative for us, to the exclusion of others. This is all perfectly natural, according to Kant, for the determination of our spatial intuition, in the shape of a hunter, that we say is delineated by these particular stars is something that is produced by our imagination. It has, in this sense, its origin in us even though it is prompted by sensation. But if we now consider, abstractly, the properties of the intuition through which one produces such an image, apart from, further, whatever particular concept (e.g., of a hunter) that it is being determined in accordance with, then this is what Kant means by the pure intuition of space.

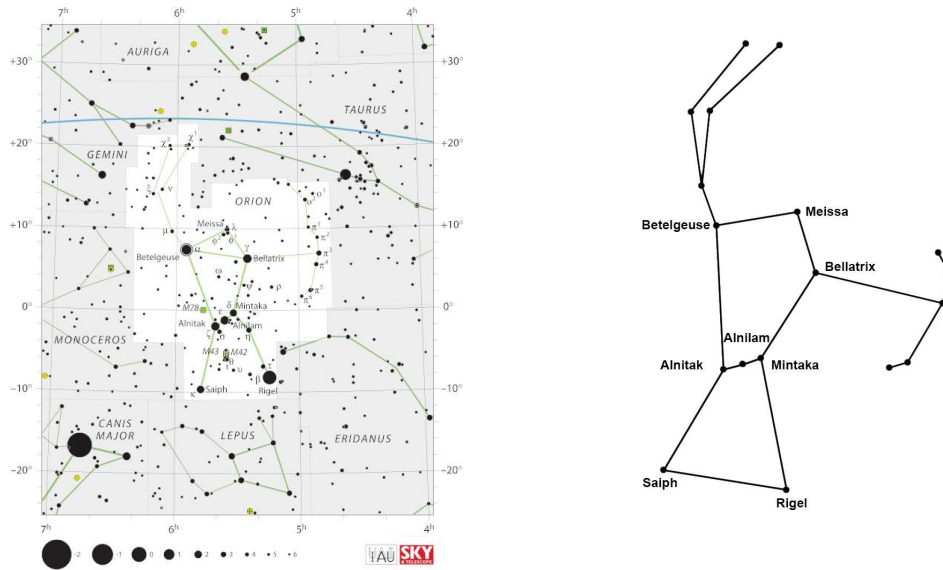


Figure 1: The constellation Orion, the hunter. Left image source: IAU and Sky & Telescope magazine (Roger Sinnott & Rick Fienberg): https://commons.wikimedia.org/wiki/File:Orion_IAU.svg. Right image source: User Mtpanchal: [https://commons.wikimedia.org/wiki/File:Stars_in_orion_constellation_\(connected\).jpg](https://commons.wikimedia.org/wiki/File:Stars_in_orion_constellation_(connected).jpg)

A number of questions remain to be clarified. Not least among them is the question of what exactly to make of what I have been calling the power of imagination. But before addressing this, in the next section we will continue with a closer look at what more Kant has to say about the source of the pure intuition of time in particular.

4 Inner intuition and our sense of ourselves

Time, as I mentioned, is the pure form of inner sense, i.e., the form in accordance with which our various mental representations are related to one another within our evolving inner state. Since no consideration is given to whether the representations so ordered ultimately concern internal or external things, time is thereby a formal condition for both inner and outer appearances.

Space, as the pure form of all outer intuitions, is limited as an *a priori* condition merely to outer intuitions. But since, on the contrary, all representations, whether or not they have outer things as their object, nevertheless as determinations of the mind themselves belong to the inner state, while this inner state belongs under the formal condition of inner intuition, and thus of time, so time is an *a priori* condition of all appearance in general, and indeed the immediate [(*unmittelbare*)] condition of the inner intuition (of our souls), and thereby also the mediate [(*mittelbar*)] condition of outer appearances (A34/B50–51).

The statement that time is the “immediate condition of the inner intuition of our souls” needs some unpacking. Out of context, one might consider it to be compatible with (and, indeed, perhaps even suggest) conceiving of time as something that, in addition, we can take to exist independently of the nature of our sensibility. This, however,

is precisely the kind of inference that is ruled out by Kant's general doctrine of *transcendental idealism*, according to which the forms through which objects appear to us are not to be taken to be the forms of things themselves. Plausibly with the goal of forestalling this kind of confusion, the B edition of the first *Critique* adds a subsection (B66–73) to the end of Kant's "General remarks on the transcendental aesthetic" (the transcendental aesthetic being the (first) part of the first *Critique* in which Kant lays out his doctrines of inner and outer sense), which includes an argument for the ideality of time. This is not Kant's only argument for the ideality of time,⁶ but his argument here is that when it comes to cognition, all that we are capable of making objective determinations about, with regard to our intuition of a thing, are the relations (of extension, motion, and force of motion) through which we describe its various appearances. Such relations alone, however, never supply us with knowledge of the nature of the things in themselves, for Kant, for "what is present in the place, or what it produces in the things themselves besides the alteration of place, is not given through these relations" (B67). In the case of outer intuition this means that ultimately all that we are capable of determining objectively are spatial relations between the object and the subject of cognition. In the case of inner sense, Kant argues that the same is true, with the issue complicated by the fact that (as we mentioned earlier) the object of inner sense is not directly intuitable but only indirectly through the intermediary of outer sense.

Before we unpack this further (in the next section), let us pause here to address a worry raised at this point by Henry Allison (Allison, 1983, pp. 258–259): How can it be appropriate to call inner sense a *sense*, Allison asks, if it has no manifold of its own—what should we take it to be a sense of? One suggestion that Allison considers, which he attributes to T. D. Weldon (1958), is that we should understand Kant's notion of inner sense to be a kind of second-order awareness of our past acts of first-order awareness (a view on the matter that Kant apparently shared with the psychologist Johann Nicholas Tetens). For Kant, according to Weldon, first-order acts of awareness always concern the objects of outer sense, while inner sense is identified with our second-order awareness of these first-order acts. It is a point in favour of Weldon's suggestion that it seems to provide a natural reading, at least on a superficial level, of Kant's claim that inner sense "does not represent anything *except insofar as something is posited* [(*gesetzt*)] in the mind" and that it "can be nothing other than the way in which the mind is affected by its own activity, namely this positing [(*Setzen*)] of its representation, thus the way it is affected through itself" (B67–68, my emphasis). Allison nevertheless rejects Weldon's reading. He points out that Kant as a rule distinguishes mere empirical consciousness from *self-consciousness*, which Allison identifies with pure apperception, and according to Allison the way that Kant does so is hard to square with Weldon's reading. Allison cites the following passage from Kant's *Anthropology*, where Kant writes that:

The inner sense is not pure apperception, a consciousness of what man does, since that belongs to the faculty of thinking. The inner sense is rather a consciousness of what man experiences, as far as he is affected by his own play of thought (Kant, 1978, §24).

Allison continues:

⁶See, for instance, A32–A36/B49–B53, and there are similar arguments regarding space, e.g., at A26–30/B42–45.

we need only note the incompatibility of this account with Weldon's suggestion that inner sense provides an awareness of past *acts* of awareness. Indeed, one of the few relatively clear aspects of Kant's theory of self-knowledge is that the consciousness of the act of thinking is assigned to apperception and not to inner sense (Allison, 1983, p. 260).

Allison is certainly right that one should not confuse pure apperception with inner sense. But perhaps there is more to be said for Weldon's general suggestion. Kant is telling us, here, that inner sense is a consciousness of what we experience *as far as we are affected by our own play of thought*, after all, which suggests a second-order relation of some kind. Further, in the A edition of his transcendental deduction of the categories of the understanding, Kant tells us that a reference to the self is implicit in empirical consciousness, explicitly contrasting "empirical apperception" with "transcendental apperception," describing the latter as "the original and necessary consciousness of the identity of oneself" (A108), and arguing that it is only in virtue of transcendental apperception that a 'standing or abiding' self is possible:

Now this original and transcendental condition is nothing other than the **transcendental apperception**. The consciousness of oneself in accordance with the determinations of our state in internal perception is merely empirical, forever variable; it can provide no standing or abiding [(*stehendes oder bleibendes*)] self in this stream of inner appearances, and is customarily called **inner sense** or **empirical apperception** [(*empirische Apperzeption*)] (A106–107).

This raises a number of further questions. Granted that (to superficially answer Allison's question regarding what inner sense is a sense of) inner sense is our sense of the way that our mind is affected by its own self-conscious activity, Kant still owes us an account of the general nature of this activity, of exactly how our minds are affected by it, and of what it can mean to be self-conscious in the absence of a standing or abiding self.

5 The nature of self-affection and the transcendental deduction of the categories

Towards the end of the B-edition transcendental aesthetic, after arguing (as we discussed above) for the ideality of time, Kant then asks us to consider a (presumably hypothetical) being that we can imagine to be capable of bringing a manifold of ("intellectual") intuition into existence, which it thereby becomes conscious of, simply by thinking. Unlike such a being, Kant writes,

In human beings this consciousness [*Bewußtsein*] requires *inner perception* [*Wahrnehmung*] of the manifold that is antecedently given in the subject, and the manner in which this is given in the mind without spontaneity must be called sensibility on account of this difference. (B68, emphasis added).

There is ample textual evidence that Kant held perception to imply a kind of self-consciousness (Dyck, 2006, 30–35). He begins the chapter of the first *Critique* dedicated to what he calls the anticipations of perception, for instance, with the definition:

“Perception is empirical consciousness, i.e., one in which there is at the same time sensation” (B207). Since sensation refers to the self, insofar as Kant identifies sensation with our capacity for representing objects (as we saw in Section 2), then with Dyck (2006, 31), I want to note that this is just another way of saying that perception, for Kant, requires self-consciousness, at least in this minimal sense.

Returning to the transcendental aesthetic, Kant next tells us that in human beings, the apprehension of what is given in the mind, or what he will later call the *synthesis of apprehension*, produces an effect upon our sensibility. In the case, specifically, of inner sense,

If the faculty for becoming conscious of oneself *is to seek out (apprehend) that which lies in the mind, it must affect the latter*, and it can only produce an intuition of itself in such a way, whose form, however, which antecedently grounds it in the mind, determines the way in which the manifold is together in the mind in the representation of time; (B68–69, emphasis added).

The problem is now to spell out just how the act of apprehension is supposed to give rise to an effect on inner sense. In connection with this, H. J. Paton notes that outer affection and self-affection differ significantly in the following way: “affection by outer objects gives us the matter of intuition, whereas the affection by ourselves does not give us a new matter, but merely combines the given matter under a new form of time” (Paton, 1936, p. 400). Allison worries, however, that this disanalogy calls Kant’s whole doctrine of self-affection into question.

The disanalogy is clearly pointed out by Paton, who remarks that the function of affection by external objects is to supply the sensible data or raw materials for knowledge, while the function of self-affection is to combine this data in consciousness in accordance with the conditions of time. The original connection that Kant asserts between affection and sensibility, however, rests upon the conception of affection as the source of sensible data; that is to say, it rests upon an understanding of affection as outer affection. Consequently, it hardly seems to follow that a comparable connection with sensibility must be assigned to self-affection. In fact, it is easy to see that the disanalogy is simply a reflection of the same difficulty that we have considered previously; namely that inner sense has no manifold of its own (Allison, 1983, p. 266).

Yet after pointing out the problem, Paton himself goes on to suggest a way to resolve it:

[Kant] mentions attention as an example of the kind of affection or influence he has in mind. This would be simple enough, if he had merely said that by attention we bring new objects or appearances before the mind, and so add to the content of inner sense, not indeed by giving something new (as is done in sensation), but by bringing into consciousness what is already given in sensation (Paton, 1936, p. 400).

To fully resolve the question we will need to have a closer look at what Kant says about the synthesis of apprehension in the first *Critique*, specifically in his section on

the transcendental deduction of the pure concepts of the understanding, or *categories*, which I now introduce. Just as space and time are the pure forms of sensibility, so the categories (three each under the headings of Quantity, Quality, Relation, and Modality) are the pure forms of thought insofar as they correspond, according to Kant, to the logical forms of judging (A79/B105) in relation to objects that can be given to us in intuition (A62/B87). We reason in accordance with the categories habitually, according to Kant, as we acquire information about the world, imagining their validity as we do so to be *a priori*, and the goal of a transcendental deduction of the categories, for Kant, is to supply us with confidence in our entitlement (*Befugnis*) (A85/B117) to use them as such. This is especially important, according to Kant, given their claim to relate to objects *a priori*, for, given this, there can be no proof of their validity on the basis of experience, or in other words there can be, according to Kant, no *empirical deduction* of their validity (A85). A *transcendental deduction* of the categories must, rather, show us how it is possible to apply them to the determination of pure intuition as such in the cognition of an object, and how, thereby, we may consider them to be universally valid forms of thought in relation to what is possible to experience.

In the A-edition version of the transcendental deduction, Kant tells us that

There are . . . three original sources (capacities or faculties of the soul), which contain the conditions for the possibility of all experience, and cannot themselves be derived from any other faculty of the mind, namely, **sense**, **imagination**, and **apperception**. On these three are grounded 1) the **synopsis** of the manifold *a priori* through sense; 2) the **synthesis** of this manifold through the imagination; finally 3) the **unity** of this synthesis through original apperception. (A94–95).

The role of the understanding, as it concerns cognition, is in general to synthesise, i.e., to combine the manifold of intuition in accordance with the rule associated with a particular concept. What Kant is relating in the above passage is that when we examine the understanding's synthesis of sensible intuition as such, we see that it in fact involves a number of conceptually distinct forms of synthesis. He elaborates:

If therefore I ascribe a synopsis to sense, because it contains a manifold in its intuition, a synthesis must always correspond to this, and **receptivity** can make cognitions possible only if combined with **spontaneity**. This is now the ground of a threefold synthesis, which is necessarily found in all cognition: that, namely, of the **apprehension** of the representations, as modifications of the mind in intuition; of the **reproduction** of them in the imagination; and of their **recognition** in the concept. Now these direct us toward three subjective sources of cognition, which make possible even the understanding and, through the latter, all experience as an empirical product of understanding. (A97–98).

Kant rewrote his transcendental deduction in its entirety for the B edition of the first *Critique*. As the B-edition transcendental deduction is where Kant's comments on the nature of self-affection and its relation to the synthesis of apprehension are, in the main, to be found, it is to it that we now turn. Beginning at B150, Kant points out that if we consider the categories as they refer merely to intuition as such, i.e., not necessarily as they refer to our own sensible forms of intuition, then from this perspective they are

401 simply to be thought of as the forms of thought in general. In the B edition, Kant calls
402 the synthesis according to the categories, taken in this sense, *intellectual synthesis*, and
403 he distinguishes it from *figurative synthesis*, which is the application of the categories
404 to sensible intuition.

405 When it concerns sensible intuition as such, the figurative synthesis is also called
406 the *transcendental synthesis of imagination*. Imagination in general, Kant tells us, “is
407 the faculty for representing an object even **without its presence** in intuition” (B151).
408 In other words, it is the power to *produce* representations spontaneously. But since the
409 imagination’s synthesis (unlike the intellectual synthesis associated with the power of
410 the understanding as such) is constrained by the pure forms of our sensible intuition,
411 which relate everything that is given to us through sensibility as such, “the imagination
412 is to this extent a faculty for determining the sensibility *a priori*” in accordance with
413 the categories (B152). This synthesis, moreover, “is an effect [*Wirkung*] of the under-
414 standing on sensibility and its first application (and at the same time the ground of all
415 others) to objects of the intuition that is possible for us” (B152).

416 Kant now writes:

417 Here is now the place to make intelligible the paradox that must have struck
418 everyone in the exposition of the form of inner sense . . . namely how this
419 presents even ourselves to consciousness only as we appear to ourselves, not
420 as we are in ourselves, since we intuit ourselves only as we are internally
421 **affected**, which seems to be contradictory, since we would have to relate to
422 ourselves passively; for this reason it is customary in the systems of psychol-
423 ogy to treat **inner sense** as the same as the faculty of **apperception** (which
424 we carefully distinguish) (B152–153).

425 Recall Allison’s criticism of Weldon’s suggestion to identify inner sense with the
426 second-order awareness of our first-order acts of awareness. Allison accused Weldon
427 of conflating inner sense with (transcendental) apperception, and at the end of this
428 passage, Kant seems to be warning us against the same kind of confusion. But just
429 as then, although Kant is here contrasting empirical apperception with transcendental
430 apperception, it is clear, in this case from the first part of the passage just quoted, that
431 there is more to be said.

432 Kant tells us, next, that “That which determines the inner sense is the understanding
433 and its original faculty of combining the manifold of intuition” (B153). This is, of
434 course, just the transcendental synthesis of the imagination that we discussed above,
435 i.e.,

436 nothing other than the unity of the action of which it is conscious as such
437 even without sensibility, but through which it is capable of itself determining
438 sensibility internally with regard to the manifold that may be given to it
439 in accordance with the form of its intuition. Under the designation of a
440 **transcendental synthesis of the imagination**, it therefore exercises that
441 action on the **passive** subject, whose **faculty** it is, about which we rightly
442 say that the inner sense is thereby affected (B153–154).

443 Recall our discussion of the example of the constellation Orion from the end of Sec-
444 tion 3. The idea here seems to be that, just as we saw in the case of outer sense, a

determination of inner sense, via the transcendental synthesis of the imagination, in accordance with its pure form, produces a determinate pure intuition of time representing the subject's effect on itself.

inner sense, [as opposed to transcendental apperception], contains [, like it,] the mere **form** of intuition, but without combination of the manifold in it, and thus it does not yet contain any **determinate** intuition at all, which is possible *only through the consciousness of the determination of the manifold through the transcendental action of the imagination* (B154, emphasis added).

He goes on:

We also always perceive this in ourselves. We cannot think of a line without **drawing** it in thought, we cannot think of a circle without **describing** it, . . . and we cannot even represent time without, in **drawing** a straight line (which is to be the external figurative representation of time), attending merely to the action of the synthesis of the manifold through which we successively determine the inner sense, and thereby attending to the succession of this determination in inner sense (B154).

The picture that Kant is giving us of the nature of inner sense and self-affection, then, looks something like this: In being presented with a manifold of outer sensible intuition as such, the understanding, first of all, acts on it in accordance with the transcendental synthesis of the imagination, which determines the manifold and yields a corresponding determinate pure intuition of space. If, now, we also attend to our action as we determine the manifold in this way, then our attention to each successive act of synthesis produces an affect upon our inner sense corresponding to each.⁷ These, in turn, may then be related to one another by the transcendental synthesis of the imagination, acting, in this case, on the manifold of inner sense. And just as before, the transcendental synthesis of the imagination yields pure intuition, in this case of the time through which the various appearances of myself are related to one another, which I can then exhibit in outer intuition by means of a line. In this way we perceive ourselves through time, with the transcendental synthesis of the imagination, in the case of inner sense, being constrained by the transcendental schematism of the categories of the understanding, which we will discuss in the next section.

6 Kant's paralogisms, transcendental schemata, and synthetic a priori principles

The other part of the first *Critique* in which Kant comments on his doctrine of inner sense is his chapter on the paralogisms of pure reason, which is another one of the parts of the first *Critique* (along with the aesthetic and deduction) that Kant amended significantly for the B edition. A paralogism is a fallacy in the form of a logical syllogism (A341/B399), and in this chapter, Kant criticises, as being guilty of this, the rational psychology of the Cartesians, according to whom it is possible to draw, from the simple

⁷For further discussion of what Kant meant by attention, see Dyck (2006, pp. 40–43).

proposition “I think”, a number of substantive claims, *a priori*, regarding the nature of the soul; namely, that the soul is a simple substance, numerically identical to itself at the various times during which it exists and interacts with the external world, the true nature of which, unlike the content of the proposition “I think” and what follows from it, we must maintain a skeptical attitude toward (A344–345 / B402–403).

Confirming our conclusion from the last section, Kant tells us, in the paralogisms’ opening section, that “I have an inner experience of this proposition [I think], which expresses the *perception of oneself* ...” (A342/B400, emphasis added). Then a little later on, he distinguishes empirical from transcendental apperception, making the point again, as he did earlier, that empirical apperception does not provide us with a standing or abiding self:

For the I is, to be sure, in all thoughts; but not the least intuition is bound up with this representation, which would distinguish it from other objects of intuition. Therefore one can, to be sure, perceive that this representation continually recurs with every thought, but not that it is a standing and abiding intuition, in which thoughts (as variable) would change. (A350).

Later in the chapter, Kant distinguishes inner from outer sense by telling us that the object of inner sense, unlike the object of outer sense, is immediately perceived:

Accordingly, in the relation of perception to its cause, it always remains doubtful whether this cause is internal or external, thus whether all so-called outer perceptions are not a mere play of our inner sense, or whether they are related to actual external objects as their cause. At least the existence of the latter is only inferred, and runs the risk of all inferences; by contrast, the object of inner sense (I myself with all my representations) is immediately perceived, and its existence suffers no doubt at all. (A368).

The determination of inner sense, in accordance with the categories, is the function of what Kant calls the categories’ *transcendental schemata*. Recall that the categories of the understanding correspond to the logical forms of judging in relation to the objects of experience as such. But although all of our empirical concepts presuppose the categories, we do not ever encounter the categories directly in intuition (something also pointed out, effectively, by Hume). A *transcendental time determination*, or in the words of the last section, the transcendental synthesis of the imagination as applied to the manifold of inner sense, provides the bridge between the category, on the one hand, and the intuition, on the other; for, insofar as it is a time *determination*, which presupposes a synthesis, it conforms to the category, and insofar as it is a *time* determination, it conforms to the intuition.

Time, as the formal condition of the manifold of inner sense, thus of the connection of all representations, contains an *a priori* manifold in pure intuition. Now a transcendental time-determination is homogeneous with the **category** (which constitutes its unity) insofar as it is **universal** and rests on a rule *a priori*. But it is on the other hand homogeneous with the **appearance** insofar as **time** is contained in every empirical representation of the manifold. Hence an application of the category to appearances becomes possible by means of the transcendental time-determination which, as the schema of the concept of the understanding, mediates the subsumption of the latter under the former (A138–139/B177–178).

The *schema* of a category is thus the rule for the application of the category in the determination of time as such. As to its source, Kant writes:

This schematism of our understanding with regard to appearances and their mere form is a hidden art in the depths of the human soul, whose true operations we can divine from nature and lay unveiled before our eyes only with difficulty. We can say only this much: the **image** is a product of the empirical faculty of productive imagination, the **schema** of sensible concepts (such as figures in space) is a product and as it were a monogram of pure *a priori* imagination, through which and in accordance with which the images first become possible, but which must be connected with the concept, to which they are in themselves never fully congruent, always only by means of the schema that they designate (A141–142/B180–181).

Thus a schema, for Kant, is a rule or procedure for constructing or exhibiting an image in intuition—as Kant writes, “The schemata are therefore nothing but *a priori* **time determinations** in accordance with rules” (A145/B185). The schema is not itself the image. Rather (see also Allison, 1983, p. 180), it corresponds to pure intuition, specifically, the transcendental schema of a category corresponds to the pure intuition of time as yielded by the application of that category to the manifold of inner sense. Thus, for instance, the schemata of the categories of quantity deal with *time series*: In attending to our apprehension of a manifold, determinate intuitions of the self are produced with each act of synthesis, the series of which is then aggregated as a way of determining the size of the object (of outer sense) apprehended. The schemata of the categories of quality deal with *time content*, i.e., with how we determine the degree of reality or intensity of what is apprehended in intuition at a given time. The schemata of relation deal with the *time order*, i.e., to the determination of the ordering of given representations in time. Finally, the schemata of modality deal with the *time sum total*, i.e., to the determination of whether a given representation exists at some, one, or at all times.

There is, perhaps, something odd in the idea that the transcendental schemata, which provide the template for the application of the categories to the manifold of inner sense—the source of the intuition of ourselves and of our inner state—are at the same time the foundation for Kant’s synthetic a priori principles: the axioms of intuition, anticipations of perception, analogies of experience, and postulates of empirical thought as such. These are, after all, the general principles for the cognition of objects and their relations in accordance with a possible experience (A148–150/B187–189). And they provide the ground for Kant’s *Metaphysical Foundations of Natural Science* (Kant, 1786), which, together with his *Critique of Teleological Judgement*, had such a great impact on the development of mathematics and natural science over the course of the 19th century and beyond (though perhaps not always in a way that Kant would have approved of or at any rate been able to anticipate). But given the “Copernican turn” characteristic of Kant’s philosophy as a whole (BXVI–XVII), this is arguably exactly what one should expect.

7 Beyond Kant

Recall that, according to Kant’s doctrine of transcendental idealism, the forms through which objects appear to us are not to be taken to be the forms of things themselves. His-

576 torically, those who have engaged with Kant's doctrine have interpreted its significance
577 in, broadly speaking, one of two different ways (Cuffaro, In press). On the one hand,
578 there are those who take its significance to be primarily ontological, i.e., to consist in
579 the positive assertion that things as they exist in themselves are not spatiotemporal and
580 thus not cognisable in accordance with Kant's synthetic a priori principles. But a natu-
581 ral objection, from this point of view, is that it does not follow, from the fact that space
582 and time are the necessary forms of our sensibility, that they cannot *also* be the forms
583 of things in themselves.⁸

584 Arguably closer to Kant himself, on the other hand, there are those, among them
585 the so-called "neo-Kantians," who take the central lesson of transcendental idealism to
586 be metaphilosophical rather than ontological, viz., that we need to adjust our standard
587 for what we take to be (mathematically certain⁹) objective cognition. In particular,
588 the central lesson, on this interpretation, is that we should abandon the traditional
589 "theocentric" conception of knowledge—the idea that to know something objectively
590 is to know it absolutely independently of the way it is apprehended—in favour of an
591 "anthropocentric" conception (this is Allison's terminology; see Allison 2004, ch. 2), i.e.,
592 such that the standard for objective cognition is reconceptualised to be that to which
593 all finite rational cognisers such as ourselves must agree. Those who interpreted Kant's
594 doctrine in this way had their own criticisms of Kant, however. These generally took
595 the form of a criticism that Kant had failed to adequately characterise the structure of
596 our cognition in some way.

597 One such thinker was Hermann von Helmholtz (1821–1894), who was at the van-
598 guard of the neo-Kantian revival of the 1850s (Beiser 2014, 6; Schnädelbach 1984,
599 103–108). In his work, Helmholtz drew on Johannes Müller's work on the physiology
600 of sense perception to argue that a number of cognitive and physiological processes
601 serve to shape our representations, so that the very same physical stimulus may be per-
602 ceived qualitatively differently depending on the specific sense organ it impinges upon.
603 Helmholtz took this to vindicate Kant (von Helmholtz, 1855, 19). Later, he was to
604 apply these and other insights into his investigations into the foundations of geometry.
605 Kant had famously argued, in the first *Critique*, that although there is nothing inconsis-
606 tent, per se, in the very idea of non-Euclidean geometry, it is nevertheless impossible to
607 construct a non-Euclidean figure via spatial intuition (A220–21/B268), and therefore,
608 impossible to come to know it as such.

609 Helmholtz was able to show (DiSalle, 2006, 128–129), however, that Euclid's first
610 four postulates presuppose the more general principle of the free mobility of a rigid
611 body, viz., that physically moving a rigid body around in space will not thereby deform
612 it.¹⁰ He then argued that the fifth, or parallel, postulate describes a global feature of

⁸The 'neglected alternative' objection is most often associated with F. A. Trendelenburg (Beiser 2014, 212–215, Gardner 1999, 107; Kanterian 2013), but the objection was also made much earlier by others, for instance by J. G. H. Feder (Sassen, 2000, 140). More recent versions of this objection are to be found in, for instance, Langton (1998, 10) and Guyer (1987, 349). Note that Kant himself does anticipate the neglected alternative objection, both in a letter to Marcus Herz of February 21, 1772 (Zweig, 1967, p. 75), as well as in the first *Critique*. At A385, for instance, he writes: "Thus there may very well be something outside us, which we call matter, corresponding to this appearance; but in the same quality as appearance it is not outside us, but is merely as a thought in us, even though this thought, through the sense just named, represents it as being found outside us."

⁹For further discussion see Dunlop (2025).

¹⁰The idea of a rigid body can be traced to Johann Friedrich Herbart (Lenoir, 2006, 151–152, 159). Euclid's first four postulates are (i) a straight line can be drawn between any two points, (ii) any straight line segment can be extended indefinitely, (iii) given a line segment, one can describe a circle with one

space that is not determined by the principle, which he now understood as a general condition on the local constructibility of figures. Helmholtz showed how the iteration of a number of basic local constructions in accordance with the principle of free mobility might result in a series of sense impressions through which one could become acquainted with the global structure of an, in general, non-Euclidean space. His analysis also highlighted the role of measurement (Lenoir, 2006, 180, 201), made precise through the concept of a rigid body, as the basis for the pre-relativistic belief that the global geometry of space is Euclidean in the first place (DiSalle, 2006, 134–136).

Another neo-Kantian thinker, Jakob Friedrich Fries (1773–1843), like Kant, saw the goal of critical philosophy to be primarily that of explicating the a priori in cognition, understood (to use Allison’s terminology) not in a theocentric but in an anthropocentric sense. But while, for Kant, the nature of our cognition is discursive, in the sense of being composed of both a sensible and a conceptual aspect—the faculties of sensibility and understanding, respectively—Fries argued that Kant’s analysis was incomplete. In particular, in the first *Critique*, in addition to sensibility and understanding, Kant also discusses the faculty of *reason*, and indeed dedicates the lion’s share of the book (the transcendental dialectic) to it. But reason, in Kant’s sense, is dialectical reason, i.e., the ability to make inferences, which for Fries is not essentially different from understanding, insofar as understanding is the faculty of judgement, and dialectical reason is merely a kind of judgement. Thus reason and understanding really represent, for Fries, only a single faculty: the faculty of mediative or reflective reasoning. Fries held that in addition to this faculty and the faculty of sensibility, there must be a third, autonomous, faculty involved in our cognition: pure reason properly speaking, which operates spontaneously and involuntarily and is, for Fries, the proper source of the a priori principles of our cognition (Leary, 1982, pp. 227–228).

These a priori principles, for Fries, are immediate, even though there is nothing in them corresponding to sensation. They are, further, cognitive, even though they are not a species of reflective cognition. They thus constitute a kind of *immediate cognition*, which when applied in particular instances manifests itself in what Fries calls a ‘feeling-for-truth’ (Nelson, 1971, p. 178–179). But because it is not reflective, this kind of cognition is not amenable to a formal proof, such as the one given by Kant on the basis of the formal conditions for the possibility of experience in the first *Critique*. Rather, it may only be ‘deduced’—i.e., discovered and described, in a functional, pragmatist, sense (Beiser 2014, 76–77; Gregory 2006, 90–91; Leary 1982, 228–229; Nelson 1971, 164–196)—through an empirical investigation of the subjective contents of our cognition.

Finally, consider Ernst Cassirer (1874–1945). Cassirer belonged to the so-called Marburg school of neo-Kantianism, whose principal figure, Cassirer’s teacher, was Hermann Cohen (1842–1918). As a rule, Marburg neo-Kantianism, like other neo-Kantian schools of that time, repudiated “psychologism,” and argued, instead, for their own particular version of what has come to be known as the epistemological (a.k.a. logical) interpretation of Kant’s philosophy. Cohen, understood intuition to be a mere abstraction, capturing the logical role played by sensibility in our cognition (Cohen, 1871, 90–91), and held there to be no distinction in kind between concepts and intuition. As for Cassirer, his own attitude towards the question can be gleaned from his commentary

endpoint as its centre and the segment’s length as its radius, and (iv) any two right angles are equal to one another.

on the puzzles of quantum mechanics, from his 1936 book on the subject:¹¹

The “crisis of causality” produced by quantum mechanics certainly persists and is quite serious. But it is a crisis not of the pure causal concept but “in the mode of viewing nature” . . . it shows us that we can no longer refer this concept, in the customary manner, to the perception of “pure time” or schematize it with the latter. (Cassirer, 1956 [1936], 164).

Then, a little later, he writes,

Transcendental logic can thus no longer be connected with or be dependent on transcendental aesthetics, as was the case in Kant’s system. The demanded specialization, indispensable for the empirical use of the causal concept, must now be looked for within the domain of concepts itself. (Cassirer, 1956 [1936], 166).

It is important that for Cassirer, dropping Kant’s transcendental aesthetic does not entail dropping his transcendental logic as well. Rather, transcendental logic is to be understood in a more general sense as a unique intellectual faculty of ‘pure synthesis’ that is no longer necessarily connected with sensibility. Thus, in his 1929 *Philosophy of Symbolic Forms*, Cassirer writes that mathematics, logic, and scientific thought in general are to be understood as specific forms of a more general expressive function of meaning, such that no one of the forms is reducible to the other, each bringing to light a distinctive aspect of reality.

Certain concepts—such as those of number, time, and space—represent, as it were, original forms of synthesis, which are indispensable wherever a “multiplicity” is to be taken together into a “unity,” wherever a manifold is to be broken down and articulated according to determinate forms. But as we have seen, this articulation is not effected in the same way in all fields: rather, its mode depends essentially on the specific structural principle which is operative and dominant in each particular sphere. Thus, in particular, language and myth each reveal a “modality,” which is specific to it and in a sense lends a common tonality to all its individual structures. (Cassirer, 1957 [1929], 13).

How does all of this relate to Gödel? Gödel was obviously not a Kantian. But in an important sense, neither was Cassirer; nor for that matter, were any of the other neo-Kantians, like the ones we just surveyed, who took themselves to have improved upon Kant’s notion of intuition in various ways. Should we then think of Gödel, like them, as some kind of neo-Kantian? Given the importance that Gödel placed on Kant in his own development as a philosopher, something that he shared with these other thinkers, one might be tempted to do so, although without qualification this would present a misleading characterisation of Gödel’s thought; for unlike these other thinkers, who

¹¹The other important neo-Kantian thinker of that period to comment on quantum mechanics was Grete Hermann (1901–1984), in her paper on the “Natural-Philosophical Foundations of Quantum Mechanics” (Hermann, 2017 [1936]). For further discussion, see Cuffaro (2022).

were fundamentally sympathetic to Kant, at least in spirit, Gödel's philosophical motivations were always fundamentally Leibnizian, or perhaps neo-Leibnizian (Yourgrau, 2024, 18).¹²

Nevertheless, there are intelligible connections to be drawn between the two thinkers, that can be brought into focus if we return to the question with which we began this chapter, namely the question of whether mathematical intuition relates to its objects analogously to the way that sense perception relates to its objects. I want to suggest that even Kant would affirm that the answer is yes, there is indeed an analogy between sense perception and mathematical intuition as conceived of by Gödel, although this does not imply, of course, that Kant would have accepted the possibility of the latter. Moreover we have just seen in the last section how various neo-Kantian thinkers have, over the ensuing decades, amended Kant's notion of intuition in various ways that at least sometimes were very productive for mathematical physics and empirical science in general. Gödel's own view can be seen as another amendment of Kant's notion of intuition of this kind, and in that sense, I think, we can call him a neo-Kantian, even if, owing to the considerations mentioned in the previous paragraph, we should be wary of taking the label very seriously.

Hao Wang tells us that

According to G, there is a close relationship between the concept of set and the categories of pure understanding in Kant's sense. [Quoting Gödel:] 'Namely, the function of both is "synthesis," i.e., the generating of unities out of manifolds (e.g., in Kant, of the idea of *one* object out of its various aspects).' 'It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective. Rather they, too, may represent an aspect of objective reality, but as opposed to sensations, their presence in us may be due to another kind of relationship between ourselves and reality' (Wang, 1987, 205).

Now, as Wang had previously explained,

According to G, a set is something obtainable from some well-defined objects by iterated application (including transfinite iteration) of the operation 'set of,' not something obtained by dividing the totality of objects into two categories ... The 'pure' sets are those sets that remain if all objects that are not sets are disregarded; or, in other words, a pure set is something obtainable from the empty set by iterated application of the operation 'set of.' ... According to G mathematics may be taken as the study of pure sets. (Wang, 1987, 194).

Note how the concept of a pure set is defined in terms of a rule-governed activity, namely, in terms of what is obtained by iterating the operation 'set of'. Recall that for Kant, similarly, pure intuition does not consist in sensible images per se. The latter, rather, are merely the product of the power of our imagination. This is especially clear in the case of time, which as we saw above, does not even have a proper manifold of its

¹²Beiser (2014, 13–16) argues that figures like Reinhold, Fichte, Schelling, and Hegel should be thought of as neo-Kantians in a similar sense, i.e., as being strongly motivated by Kant despite fundamentally disagreeing with him.

own. And yet, as Gödel understood very well, for Kant, pure intuition is always related to sensibility in the sense that the images produced by our imagination, by means of pure intuition, are necessarily sensible images, even in the case where what is being exhibited is a temporal interval. There is no analogous requirement of what one might call visualisability, in the case of mathematical intuition, for Gödel.

As for the neo-Kantians that we discussed in this section, Cassirer's is, perhaps, the most similar to Gödel's insofar as Cassirer explicitly renounces the stark divide that Kant wants to draw between concepts and intuition.¹³ But there are also, undoubtedly, interesting conceptual connections to be drawn between Gödel's conjecture that "some physical organ is necessary to make the handling of abstract impressions (as opposed to sense impressions) possible" (Wang, 1987, 190), and the kind of immediate cognition that Fries attempted to capture via his faculty of "feeling-for-truth." But I will leave these topics for others to explore, or, at any rate, for another occasion.

8 Summary

We saw, in Section 2, that according to Kant, space and time are the pure forms of outer and inner sense, respectively, which respectively correspond to our capacity to represent objects as being outside ourselves, and our capacity to represent anything at all, the latter being the source of the intuition of our self and of our own inner state. We then saw, in Section 3, how the pure forms of sense can also, according to Kant, be thought of as pure intuition, and how Kant distinguishes pure intuition from the images that are exhibited through it by our power of imagination. In Section 4 we restricted our focus to inner sense. I explained that, for Kant, we intuit ourselves, through inner sense, only as we appear to ourselves, not as we are. In order to further clarify this, in Section 5, we then turned to Kant's transcendental deduction of the categories of the understanding, to flesh out what it means to appear to oneself, and we saw that any combination of the manifold of intuition by our general faculty of understanding, involves what Kant calls the synthesis of apprehension of what is given in sensibility (i.e., given without spontaneity). We further saw how, on Kant's view, attending to this act of synthesis produces an effect upon our inner sense, along with a corresponding determinate intuition that is related, in time, to the other determinate intuitions that we have of ourselves and of our own inner state. We saw that, in this way, we perceive ourselves through time. In Section 6 we turned to Kant's paralogisms of pure reason, and then to his discussion of the schematism of the pure concepts of the understanding, and I explained how Kant's doctrine of inner sense underlies his philosophy of mathematics and the exact natural sciences. Then, in Section 7, we discussed the broader influence of Kant's ideas. We saw how various thinkers of the nineteenth and early twentieth centuries took themselves to have improved on Kant's understanding of the nature of intuition in various ways. Finally, we connected this back to Gödel. We concluded that there is indeed an analogy between perception, in Kant's sense, and mathematical intuition in Gödel's sense, though the viability of Gödel's conception of the latter is a separate question.

¹³For a commentary on some of the conceptual connections between Cassirer and Gödel, see Dorato (2002).

References

- Allison, H. E. (1983). *Kant's Transcendental Idealism, An Interpretation and Defense*. New Haven: Yale University Press.
- Allison, H. E. (2004). *Kant's Transcendental Idealism, An Interpretation and Defense*. New Haven: Yale University Press, revised and enlarged ed.
- Beiser, F. C. (2014). *The Genesis of Neo-Kantianism*. Oxford: Oxford University Press.
- Cassirer, E. (1956 [1936]). *Determinism and Indeterminism in Modern Physics*. Trans. O. T. Benfey. New Haven: Yale University Press.
- Cassirer, E. (1957 [1929]). *The Philosophy of Symbolic Forms, Volume 3: The Phenomenology of Knowledge*. Fredericksburg: Yale University Press.
- Cohen, H. (1871). *Kants Theorie der Erfahrung*. Berlin: Dümmler.
- Cuffaro, M. E. (2022). Grete Hermann, quantum mechanics, and the evolution of Kantian philosophy. In J. Peijnenburg, & A. A. Verhaegh (Eds.) *Women in the History of Analytic Philosophy*, (pp. 113–145). Dordrecht: Springer.
- Cuffaro, M. E. (In press). Transcendental idealism and its influence on nineteenth century science. In E. Crull, & E. Peterson (Eds.) *History and Philosophy of Modern Science: 1750-1900*. Bloomsbury.
- DiSalle, R. (2006). *Understanding Space-time*. Cambridge: Cambridge University Press.
- Dorato, M. (2002). Kant, gödel and relativity. In P. Gardenfors, K. Kijania-Placek, & J. Wolenski (Eds.) *Proceedings of the Invited Papers for the 11th International Congress of the Logic and Methodology of Science*, (pp. 329–346). Dordrecht: Synthese.
- Dunlop, K. (2025). Mathematics: Systematic unity and construction in the theory of conic sections. In G. Gava, T. Sturm, & A. Vesper (Eds.) *Kant and the Systematicity of the Sciences*, (pp. 114–147). New York: Routledge.
- Dyck, C. W. (2006). Empirical consciousness explained: Self-affection, (self-)consciousness and perception in the B deduction. *Kantian Review*, 11, 29–53.
- Dyck, C. W. (2016). The scope of inner sense: The development of kant's psychology in the silent decade. *Con-Textos Kantianos*, 3, 326–344.
- Friedman, M., & Nordmann, A. (Eds.) (2006). *The Kantian Legacy in 19th Century Science*. Cambridge: MIT Press.
- Gardner, S. (1999). *Routledge Philosophy Guidebook to Kant and the Critique of Pure Reason*. London: Routledge.
- Gödel, K. (1983). What is Cantor's continuum problem? In P. Benacerraf, & H. Putnam (Eds.) *Philosophy of Mathematics: Selected Readings*, (pp. 470–485). Cambridge: Cambridge University Press, second ed. Expanded and revised for second edition.

- 814 Gödel, K. (1995). The modern development of the foundations of mathematics in the
815 light of philosophy. In S. Feferman, J. W. D. Jr., W. Goldfarb, C. Parsons, & R. N.
816 Solovay (Eds.) *Collected Works, Volume III: Unpublished essays and lectures*, (pp. 403–
817 404). Oxford: Oxford University Press.
- 818 Gregory, F. (2006). Extending Kant: The origins and nature of Jakob Friedrich Fries's
819 philosophy of science. In (Friedman & Nordmann, 2006, pp. 81–100).
- 820 Guyer, P. (1987). *Kant and the Claims of Knowledge*. Cambridge: Cambridge University
821 Press.
- 822 Harper, W. (1984). Kant's empirical realism and the distinction between subjective and
823 objective succession. In Harper, & Meerbote (Eds.) *Kant on Causality, Freedom and*
824 *Objectivity*, (pp. 108–137). Minneapolis: University of Minnesota Press.
- 825 Hermann, G. (2017 [1936]). Natural-philosophical foundations of quantum mechanics
826 (1935). Trans. E. Crull, & G. Bacciagaluppi. In E. Crull, & G. Bacciagaluppi (Eds.)
827 *Grete Hermann: Between Physics and Philosophy*, (pp. 239–278). Dordrecht: Springer.
- 828 Indregard, J. J. (2017). Self-affection and pure intuition in kant. *Australasian Journal*
829 *of Philosophy*, 95, 627–643.
- 830 Kant, I. (1781). *Kritik der Reinen Vernunft*. Hamburg: Felix Meiner. Reprinted: 1998.
- 831 Kant, I. (1786). *Metaphysische Anfangsgründe der Naturwissenschaft*. Translation:
832 Michael Friedman, *Metaphysical Foundations of Natural Science*, Cambridge: Cam-
833 bridge University Press, 2004.
- 834 Kant, I. (1978). *Anthropology From a Pragmatic Point of View*. Trans. V. L. Dowdell.
835 Carbondale: Southern Illinois University Press.
- 836 Kant, I. (1998). *Critique of Pure Reason*. Trans. P. Guyer, & A. W. Wood. Cambridge:
837 Cambridge University Press.
- 838 Kanterian, E. (2013). The ideality of space and time: Trendelenburg versus Kant, Fis-
839 cher and Bird. *Kantian Review*, 18, 263—288.
- 840 Langton, R. (1998). *Kantian Humility*. Oxford: Oxford University Press.
- 841 Leary, D. E. (1982). The psychology of Jakob Friedrich Fries (1773–1843): Its context,
842 nature, and historical significance. *Storia E Critica Della Psicologia*, 3, 217–248.
- 843 Lenoir, T. (2006). Operationalizing Kant: Manifolds, models, and mathematics in
844 Helmholtz's theories of perception. In (Friedman & Nordmann, 2006, pp. 141–210).
- 845 Martin, D. A. (2005). Gödel's conceptual realism. *The Bulletin of Symbolic Logic*, 11,
846 207–224.
- 847 Nelson, L. (1971). *Progress and Regress in Philosophy, Volume II*. Trans. H. Palmer.
848 Oxford: Basil Blackwell.
- 849 Paton, H. J. (1936). *Kant's Metaphysic of Experience, In Two Volumes*. London: George
850 Allen & Unwin Ltd.

- 851 Sassen, B. (Ed.) (2000). *Kant's Early Critics*. Trans. B. Sassen. Cambridge: Cambridge
852 University Press.
- 853 Schnädelbach, H. (1984). *Philosophy in Germany 1831–1933*. Cambridge: Cambridge
854 University Press.
- 855 Valaris, M. (2008). Inner sense, self-affection, and temporal consciousness in kant's
856 critique of pure reason. *Philosophers' Imprint*, 8, 1–18.
- 857 von Helmholtz, H. (1855). *Ueber das Sehen des Menschen*. Leipzig: Leopold Voss.
- 858 Wang, H. (1987). *Reflections on Kurt Gödel*. Cambridge: The MIT Press.
- 859 Weldon, T. D. (1958). *Kant's Critique of Pure Reason*. Oxford: Clarendon Press.
- 860 Yourgrau, P. (2024). Does reason have limits? review essay: Stephen Budiansky, jour-
861 ney to the edge of reason: The life of Kurt Gödel. *Annals of Mathematics and Philoso-*
862 *phy*, 2, 1–31.
- 863 Zweig, A. (Ed.) (1967). *Kant, Philosophical Correspondance 1759-99*. Trans. A. Zweig.
864 Chicago: The University of Chicago Press.