Kant on Pure Intuition, Inner Sense, Self-Affection, and Time, and Gödel on Kant

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Gödel on "mathematical intuition":

> "[D]espite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e. in mathematical intuition, than in sense perception." (Gödel, 1983, p. 483–484; quoted in Martin, 2005, p. 207).

These and other similar remarks by Gödel have left many of his commentators bewildered, or at any rate dissatisfied.

A questionnaire (see Wang, 1987, pp. 19–20):

> **Q** (Burke D. Grandjean, 1974): "How much importance, if any, do you attribute to each of the scholars in the above list [Ludwig Boltzmann, Jan Brouwer, Paul Finsler, Kant, Karl Kraus, Fritz Mauthner, Jules Richard, and Ludwig Wittgenstein], in the development of your interests?"
>
> **A** (Gödel): "Only Kant was imp[ortant]."

Further elaboration included in Gödel's written responses:

- "As to [your question] I would like to say that only Kant had some infl[uence] on my phil[osophical] thinking in gen[eral] & that I got acq[uainted] with him about 1922."
- "None of the scholars mentioned ... in your questionnaire, except perhaps Kant had any direct influence on the development of my interests."

Gödel's engagements with Kant's ideas in mathematics and physics continued throughout his life (Wang, 1987, pp. 27–28, 182–185, 205–208).

From an unpublished manuscript (roughly 1961):

> "But now, if the misunderstood Kant has already led to so much that is interesting in philosophy, ... how much more can we expect from Kant understood correctly?"
> (Gödel, 1995, p. 387).

Goals of this talk:

1. To elaborate Kant's doctrines of outer and inner sense, his doctrine of self-affection, and his conception of the pure intuitions of space and especially time.
2. To consider Gödel's views on mathematical intuition in this light.

Outline:

1. Sensibility and its pure forms
2. Space and time as pure intuition
3. Inner sense and self-affection
4. Transcendental schemata and synthetic a priori principles
5. Kant and Gödel

Kant on Hume (1783):

"The question was not whether the concept of cause was right, useful, and even indispensable for our knowledge of nature, for this Hume had never doubted; but whether that concept could be thought by reason *a priori*, and consequently whether it possessed an inner truth, independent of all experience ... This was Hume's problem. It was a question concerning the *origin* of the concept, not concerning its indispensability in use. Were the former decided, the conditions of its use and the sphere of its valid application would have been determined as a matter of course." (Kant, 2001, 4:258–259).

Kant's new insight:

- Metaphysical cognition (if it were possible) would be a kind of synthetic a priori cognition.

Synthetic a priori cognition:

- Cognition of distinct concepts as being necessarily connected
- Mathematical cognition is also synthetic a priori.
- How it is possible: The forms of possible experience are known a priori and are also synthetic (since appearances are ordered through them).
- Limits: synthetic a priori cognition is only possible in relation to possible experience.

"In the solution of the above problem there is at the same time contained the possibility of the pure use of reason in the grounding and execution of all sciences that contain a theoretical *a priori* cognition of objects" (B20).

(i.e., the exact, or mathematical, sciences)

Kant's discursivity thesis:

"Thoughts without content are empty; intuitions without concepts are blind. It is thus just as necessary to make the mind's concepts sensible (i.e., to add an object to them in intuition) as it is necessary to make intuitions understandable (i.e., to bring them under concepts). Further, these two faculties or capacities cannot exchange their functions. The understanding is not capable of intuiting anything, and the senses are not capable of thinking anything. Only from their unification can cognition arise." (A51/B75–76).

Kant's framework for theoretical cognition:

I. The pure forms of sensible intuition

II. The pure forms of thought

Outer sense:

- The property of our mind that enables us to represent objects as being outside ourselves (A22/B36–37).
- Formal condition: Objects are represented as being in space.
    - Space: a pure form, i.e., valid irrespective of
        - Whatever particular sensation occasions a given appearance.
        - Whatever particular concept is being used to determine a given appearance.

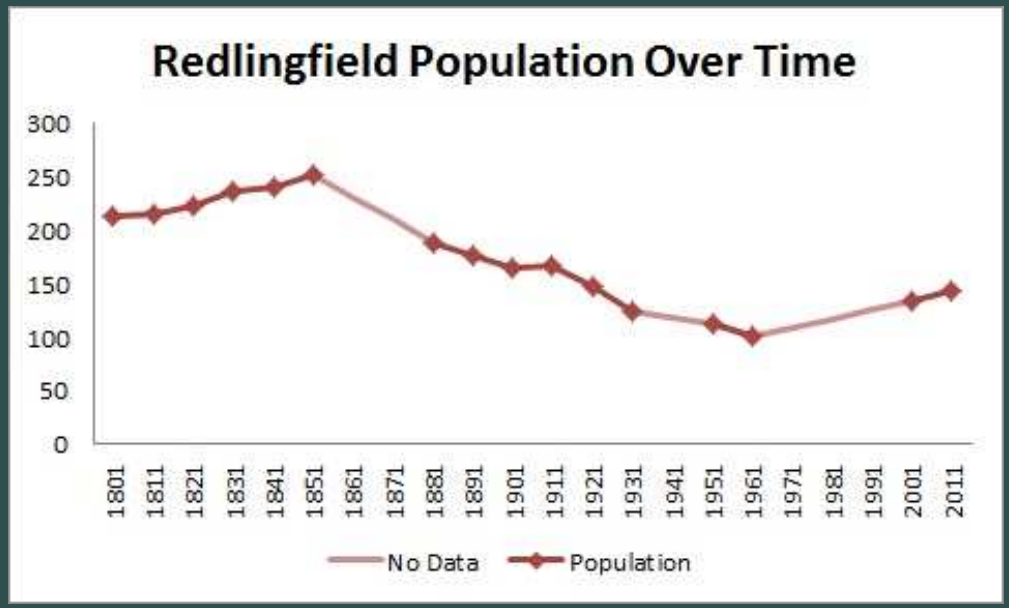
Inner sense:

- The property of our mind that enables us to represent anything at all.
- The source of the intuition of our self and of our inner state.
- Formal condition: All representations are related to one another in time.
- Time (unlike space) is not directly intuitable, but requires space as an intermediary. I.e., temporal relations are expressible by placing temporal parts into one-one correspondence with the (spatial) parts of a line.

"[We] represent the temporal sequence through a line progressing to infinity, in which the manifold constitutes a series that is of only one dimension, and infer from the properties of this line to all the properties of time, with the sole difference that the parts of the former are simultaneous but those of the latter always exist successively." From this it is also apparent that the representation of time is itself an intuition, since all its relations can be expressed in outer intuition." (A33/B50).

Unpacking this:

- (As Kant had argued earlier): Space is intuitive in the specific sense that as an infinite, given, magnitude, it cannot be captured by any particular (discursive) concept (A25/B39–40).
- Thus, that parts of time can be put into correspondence with parts of space means time is intuitive in the same sense.



However, our intuition of time is only indirect:

- Representing time as a line yields (outer) intuition, but this is not the intuition of time.
- Unlike the temporal parts of a particular history, a line drawn on paper (or on anything else) does not have a temporal beginning or ending but exists for us all at once (see also Allison, 1983, p. 256).

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"So if I separate from the representation of a body that which the understanding thinks about it, such as substance, force, divisibility, etc., as well as that which belongs to sensation, such as impenetrability, hardness, color, etc., something from this empirical intuition is still left for me, namely extension and form." These belong to the pure intuition, which occurs *a priori*, even without an actual object of the senses or sensation, as a mere form of sensibility in the mind." (A20–21/B35).

- Kant's point is not that there are things called pure intuitions existing in the mind independently of the understanding and sensibility.
- It is that even if we accept that spatial and temporal relations require (representations of) things to relate, we can still consider what is left after removing, in thought, how a given particular is sensed and understood in the context of a given experience.

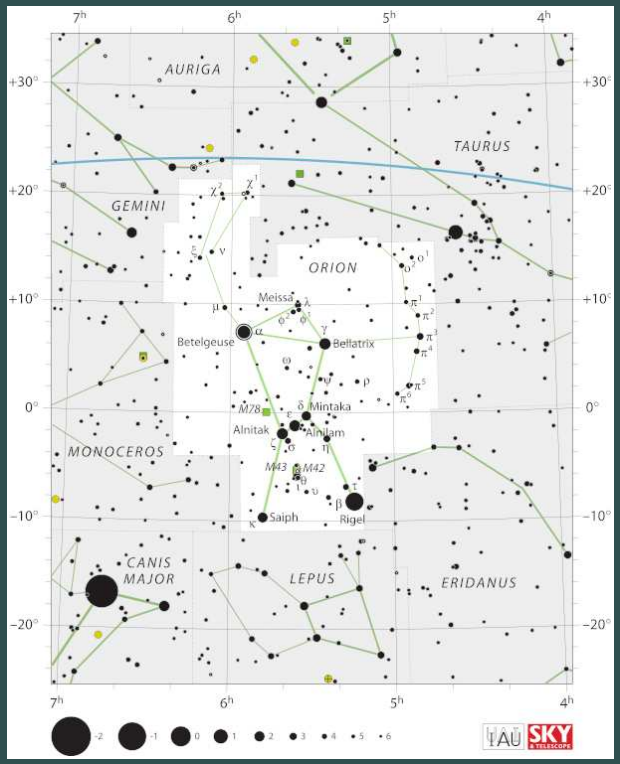
- Moreover, via our power of imagination (more on this in a bit), we are able to reason about the properties of pure intuition even when nothing is actually given to us in sensation.
- But regardless whether we reason about the properties of pure intuition on paper or in our imagination, the pure intuition of space and time can never be presented to the mind without accompanying content of some kind.

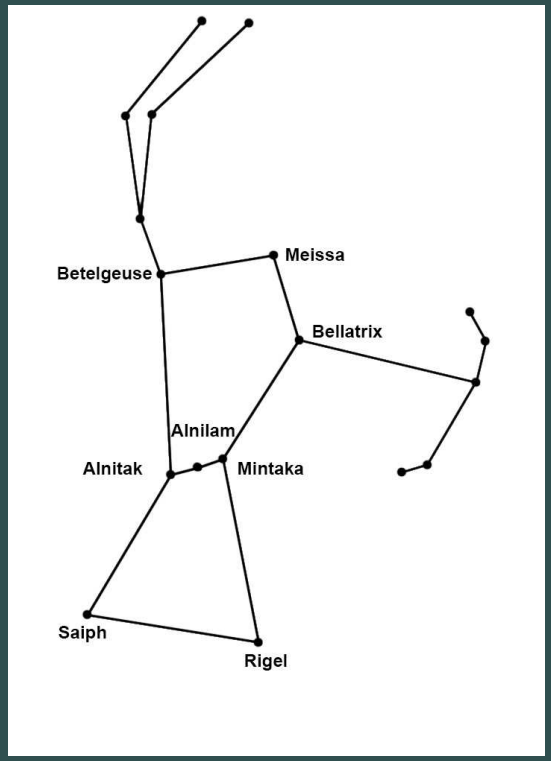
"[I]ndeed, [space and time's] representation is a mere schema, which is always related to the reproductive imagination that calls forth the objects of experience, without which they would have no significance; and thus it is with all concepts without distinction." (A156/B195).

Example:

- Imagining a square, even abstractly, requires that we imagine some thing that encloses this space (i.e., four equal-length lines drawn at right angles in the usual way).
- Importantly, however, the pure intuition of a square does not correspond to the image.

- If I could hook my brain up to a computer in order to see the image produced by my mind as I imagine a square in the abstract, I might find it far from perfect.
- This has no bearing on the content of the pure intuition underlying the image, however:
  - Indeed it is through pure intuition (called forth upon drawing a grid on the screen, for instance) that I am able to determine that my imagined square was not perfect.





- Example: The shape of the constellation Orion is not a property of the stars themselves. Moreover it is misleading, emphasising certain spatial relations and downplaying others.
- This is perfectly natural, for Kant, since the image has its origin in us (even though it is prompted by sensation).
- The pure intuition underlying the image is made manifest by considering the constructions that are required to produce it.

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Recall:

- The objects of outer sense are directly intuitable.
- The object of inner sense is not. It is only intuitable indirectly through the intermediary of space.

Objection (Allison, 1983, pp. 258–259):

- How can it be appropriate to call inner sense a sense if it has no manifold of its own – what should we take it to be a sense of?

Suggestion (Weldon, 1958):

- Inner sense is a kind of second-order awareness of past acts of first-order awareness, where the latter concern the objects of outer sense. (This was also the view of the psychologist Johann Nicholas Tetens.)

"[Inner sense] does not represent anything except insofar as something is posited [(*gesetz*)] in the mind, ... [it] can be nothing other than the way in which the mind is affected by its own activity, namely this positing [(*Setzen*)] of its representation, thus the way it is affected through itself" (B67–68).

Allison objects: Kant as a rule distinguishes mere empirical consciousness from self-consciousness (or apperception):

"The inner sense is not pure apperception, a consciousness of what man does, since that belongs to the faculty of thinking. The inner sense is rather a consciousness of what man experiences, as far as he is affected by his own play of thought." (Kant, 1798, §24).

"[W]e need only note the incompatibility of this account with Weldon's suggestion that inner sense provides an awareness of past *acts* of awareness. Indeed, one of the few relatively clear aspects of Kant's theory of self-knowledge is that the consciousness of the act of thinking is assigned to apperception and not to inner sense." (Allison, 1983, p. 260).

But this is a little too quick:

- Allison is correct to warn against conflating pure apperception with inner sense.
- But in Weldon's defence, that inner sense is a consciousness of what we experience as far as we are affected by our own play of thought seems to suggest a second-order relation.
- Further, in the Critique Kant distinguishes empirical apperception (or self-consciousness) with transcendental apperception:

"Now this original and transcendental condition is nothing other than the **transcendental apperception**. The consciousness of oneself in accordance with the determinations of our state in internal perception is merely empirical, forever variable; it can provide no standing or abiding [(*stehendes oder bleibendes*)] self in this stream of inner appearances, and is customarily called **inner sense** or **empirical apperception** [(*empirische Apperzeption*)]." (A106–107).

A thought experiment: Imagine a being that would be capable of bringing a manifold of ("intellectual") intuition into existence, that it thereby becomes conscious of, simply by thinking.

Unlike such a being,

"In human beings this consciousness [*Bewußtsein*] requires inner perception [*Wahrnehmung*] of the manifold that is antecedently given in the subject, and the manner in which this is given in the mind without spontaneity must be called sensibility on account of this difference." (B68).

Kant on perception:

- "Perception is empirical consciousness, i.e., one in which there is at the same time sensation" (B207).
- Note that sensation, for Kant, involves a reference to the self insofar as he identifies it with our capacity for representing objects.

Upshot: *Pace* Allison, both empirical and transcendental consciousness involve a reference to the self, but it is only in virtue of the latter that a "standing or abiding" self is possible.

Still to be clarified:

- Granted that (to answer Allison's question) inner sense is our sense of the way that our mind is affected by its own self-conscious activity,
- Kant still owes us an account of the general nature of this activity, of exactly how our minds are affected by it, and of what it can mean to be self-conscious in the absence of a standing or abiding self.

Kant on self-affection:

"If the faculty for becoming conscious of oneself is to seek out (apprehend) that which lies in the mind, it must affect the latter, and it can only produce an intuition of itself in such a way,"

But how, exactly, does it work?

H. J. Paton:

"[Kant] mentions attention as an example of the kind of affection or influence he has in mind. This would be simple enough, if he had merely said that by attention we bring new objects or appearances before the mind, and so add to the content of inner sense, not indeed by giving something new (as is done in sensation), but by bringing into consciousness what is already given in sensation." (Paton, 1936, p. 400).

The Categories:

- The function of our understanding is in general to synthesize the manifold of intuition in accordance with concepts.
- The categories (or pure concepts): Just as space and time are the pure forms of sensibility, so the categories are the pure forms of thought – they correspond to the logical forms of judging (A79/B105) in relation to objects that can be given to us in intuition (A62/B87).

Different types of syntheses:

- Intellectual synthesis: Application of a category in the synthesis of intuition as such (i.e., not necessarily ours).
- Figurative synthesis: Application of a category as it pertains to the synthesis of sensible intuition.
- Transcendental synthesis of imagination: Figurative synthesis as it concerns sensible intuition as such.

Imagination:

- In general, imagination (as we saw earlier) "is the faculty for representing an object even **without its presence** in intuition" (B151). I.e., it is the power to produce representations spontaneously.
- Since the imagination's synthesis is constrained (only) by the pure forms of sensible intuition as such, it is "to this extent a faculty for determining the sensibility *a priori*." (B152).

"Under the designation of a **transcendental synthesis of the imagination**, it therefore exercises that action on the **passive** subject, whose **faculty** it is, about which we rightly say that the inner sense is thereby affected." (B153–154).

- Kant's point seems to be that, just as it was in the case of the determination of outer sense (recall the example of Orion), a determination of inner sense produces a determinate pure intuition of time, in this case representing the subject's effect on itself.

"Inner sense contains the mere **form** of intuition, but without combination of the manifold in it, and thus it does not yet contain any **determinate** intuition at all, which is possible only through the consciousness of the determination of the manifold through the transcendental action of the imagination." (B154).

"We also always perceive this in ourselves. We cannot think of a line without **drawing** it in thought, we cannot think of a circle without **describing** it," ... and we cannot even represent time without, in **drawing** a straight line (which is to be the external figurative representation of time), attending merely to the action of the synthesis of the manifold through which we successively determine the inner sense, and thereby attending to the succession of this determination in inner sense." (B154).

In a nutshell:

- When presented with a manifold of outer sensible intuition as such, the understanding, first of all, acts on it in accordance with the transcendental synthesis of the imagination, which yields a corresponding determinate pure intuition of space.
- If we now also attend to our action as we determine the manifold in this way, then the attention to each successive act of synthesis produces an effect upon our inner sense corresponding to each.
- These, in turn, are then related to one another by the transcendental synthesis of the imagination, acting, in this case, on the manifold of inner sense.
- Just as before, the transcendental synthesis of the imagination yields pure intuition, in this case of the time through which the various appearances of myself are related to one another, which I can then exhibit in outer intuition by means of a line.
- In this way we perceive ourselves through time.

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An important distinction between inner and outer sense:

"[I]n the relation of perception to its cause, it always remains doubtful whether this cause is internal or external, thus whether all so-called outer perceptions are not a mere play of our inner sense, or whether they are related to actual external objects as their cause. At least the existence of the latter is only inferred, and runs the risk of all inferences;" by contrast, the object of inner sense (I myself with all my representations) is immediately perceived, and its existence suffers no doubt at all." (A368).

Inner sense is the key, for Kant, to solving the problem of the applicability of the categories of the understanding to intuition.

Consider:

- Problem of the applicability of the categories: Although our empirical concepts presuppose them, we don't ever directly encounter the categories in intuition (cf. Hume's skepticism about, e.g., causality).
- Solution: A transcendental time determination:
  - has something in common with the categories (because determination presupposes synthesis)
  - has something in common with the intuition (because it is a time determination)

Kant's transcendental schemata:

- Schema for a category: The rule for the application of the category in the determination of time as such (A137-147/B176-B187).
- Categories and their characteristic schemata:
  - of Quantity: Schemata deal with time series through which we determine the size of something.
  - of Quality: Schemata deal with time content, i.e. with how we determine the degree of reality or intensity of what is apprehended in intuition at a given time.
  - of Relation: Schemata deal with time order, i.e., to the determination of the ordering of given perceptions in time.
  - of Modality: Schemata deal with time sum total, i.e., to the determination of whether a given object exists at some, one, or all times.

Synthetic a priori principles:

- General principles governing the use of the categories in accordance with their characteristic schema.
- Also (thereby) the general principles for the cognition of objects and their relations in accordance with a possible experience.

It might seem odd that the transcendental schemata, which provide the template for the application of the categories to the manifold of inner sense – the source of the intuition of ourselves and of our inner state – are at the same time the foundation for Kant's synthetic a priori principles, and thereby for his entire philosophy of natural science.

But given the "Copernican turn" characteristic of Kant's philosophy as a whole (BXVI–XVII), this is arguably exactly what one should expect.

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"[D]espite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e. in mathematical intuition, than in sense perception."

What I want to suggest:

- There is indeed an analogy (which even Kant would affirm) between sense perception, as he understood it, and mathematical intuition as conceived of by Gödel (even if Kant would have denied the possibility of the latter).
- After Kant, many (most) neo-Kantian thinkers amended Kant's doctrine of sensibility, and in some cases rethought the very distinction between sensibility and understanding.
- Gödel was not a neo-Kantian (no more than, e.g., Hegel or Schelling at any rate), but his conception of mathematical intuition should arguably be seen in this light.

Hao Wang on Gödel on Kant:

"According to G, there is a close relationship between the concept of set and the categories of pure understanding in Kant's sense. [Quoting Gödel:] 'Namely, the function of both is "synthesis," i.e., the generating of unities out of manifolds (e.g., in Kant, of the idea of *one* object out of its various aspects).'" 'It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective. Rather they, too, may represent an aspect of objective reality, but as opposed to sensations, their presence in us may be due to another kind of relationship between ourselves and reality'." (Wang, 1987, 205).

Neo-Kantians:

- Jakob Friedrich Fries: Posited a separate faculty of reason — a kind of "feeling for truth" that he thought was required to overcome the limitations of Kant's conception of understanding as a general faculty of judgement.
    - Cf. Gödel's conjecture that "some physical organ is necessary to make the handling of abstract impressions (as opposed to sense impressions) possible" (Wang, 1987, 190).
- Hermann von Helmholtz: Revised Kant's doctrine of sensibility in the light of new results from physiology and psychology, leading to new insights in geometry.
- Hermann Cohen and Ernst Cassirer: Collapsed the distinction between sensibility and understanding, allowing for the former (now understood as ultimately conceptual) to be revised and refined over time as science progresses.

Gödel on the concept of a pure set:

"According to G, a set is something obtainable from some well-defined objects by iterated application (including transfinite iteration) of the operation 'set of,' not something obtained by dividing the totality of objects into two categories" ... The 'pure' sets are those sets that remain if all objects that are not sets are disregarded; or, in other words, a pure set is something obtainable from the empty set by iterated application of the operation 'set of.' ... According to G mathematics may be taken as the study of pure sets." (Wang, 1987, 194).

- Note how the concept of a pure set is defined in terms of a rule-governed activity, namely, in terms of what is obtained by iterating the operation 'set of'.
- Recall that for Kant, similarly, pure intuition does not consist in sensible images per se. The latter, rather, are merely the product of the power of our imagination.
    - This is especially clear in the case of time, which does not even have a proper manifold of its own.
- And yet, as Gödel understood very well, for Kant, pure intuition is always related to sensibility in the sense that the images produced by our imagination, by means of pure intuition, are necessarily sensible images, even in the case where what is being exhibited is a temporal interval.
- There is no analogous requirement of what one might call visualisability, in the case of mathematical intuition, for Gödel.

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